

# Robotic manipulators in interventional medicine and surgery

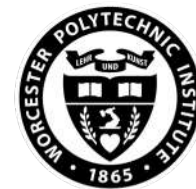
**Loris Fichera**

Cognitive Medical Technology Lab

Robotics Engineering Program

Department of Computer Science

Worcester Polytechnic Institute



**WPI**

*COMET Lab*

# Loris Fichera, Ph.D.

Assistant Professor  
in Computer Science, Robotics Engineering



## Academic Background:

Postdoc in Mechanical Engineering, Vanderbilt University - 2017

PhD in Robotics, Cognition and Interaction Technologies, IIT - 2015

BS/MS in Computer Engineering, University of Catania – 2011



My high school math teacher



Liceo Scientifico Stanislao  
Cannizzaro, Vittoria RG  
Class of 2005

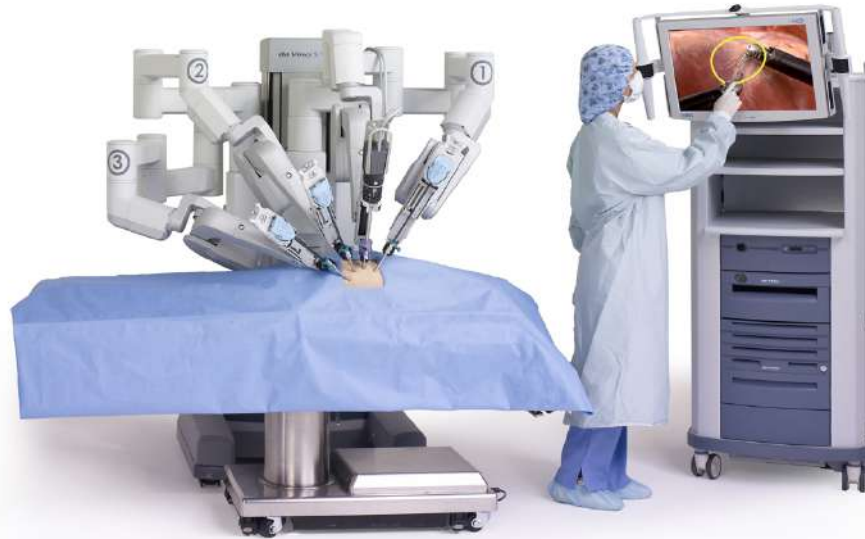


<https://www.youtube.com/watch?v=sRrnW8uga4k>

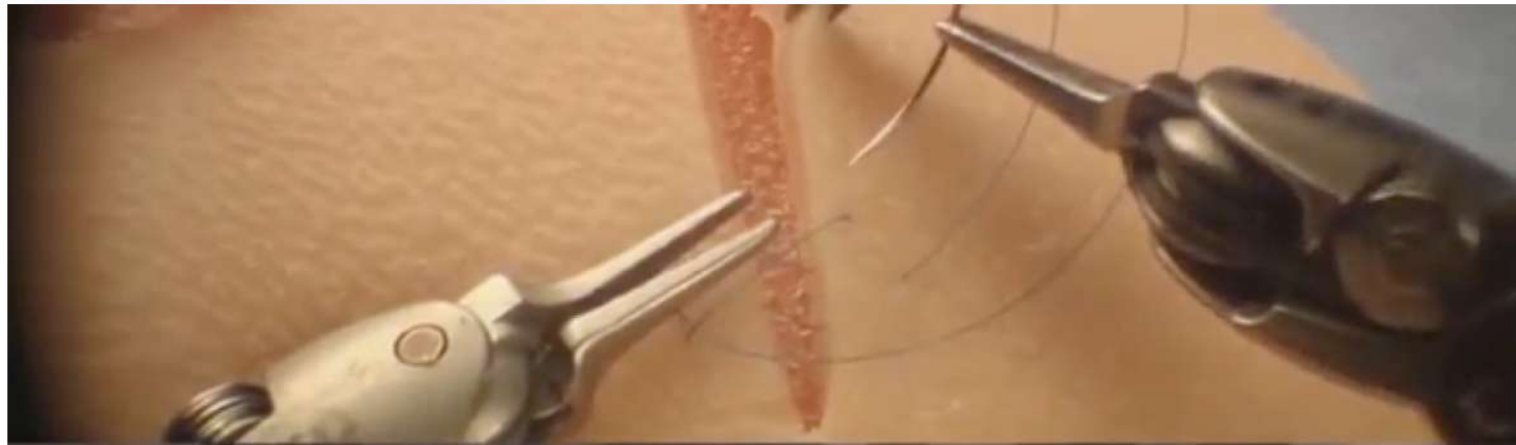
1011948 rA

<https://www.youtube.com/watch?v=0XdC1HUp-rU>

# Robotic technology today



da Vinci surgical System  
(© Intuitive Surgical, Inc.)



1006242 rA

# The first surgical robot



Credit: Roger Ressmeyer/CORBIS

# Robot kinematics

**Goal:** describe the *pose* of the robot (position and orientation)

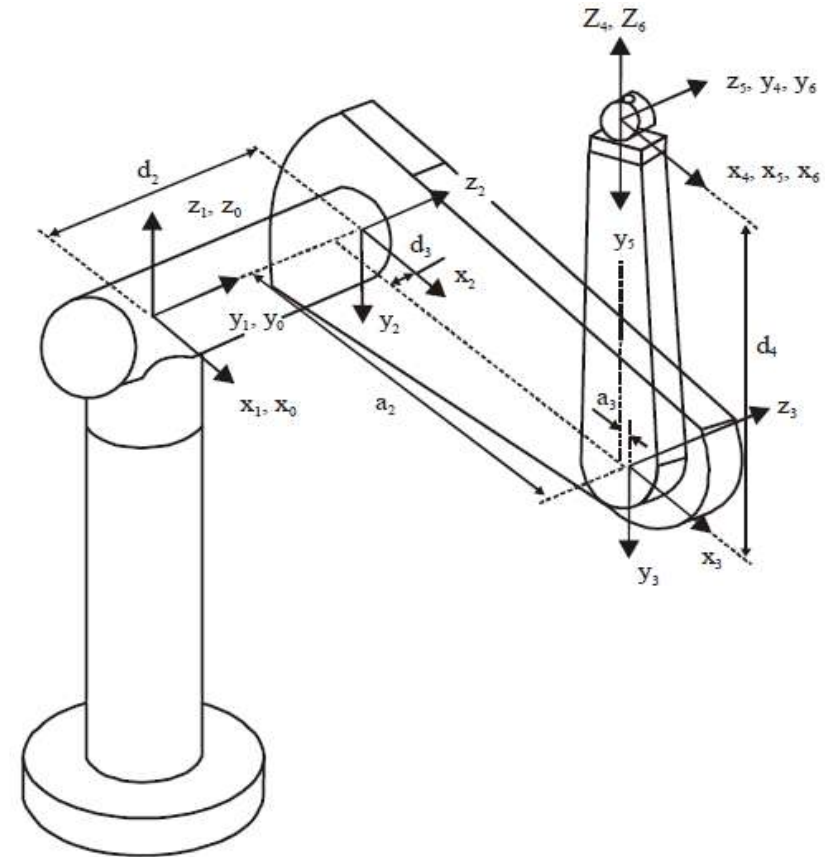


Fig. 1: PUMA 560 in the zero position with attached coordinates frames shown<sup>17</sup>



# Robot kinematics

**Goal:** describe the *pose* of the robot (position and orientation)

Transformation matrix between two consecutive joints:

$$T_i^{i+1} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation between robot base and end effector:

$$T_0^6 = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5 T_5^6$$

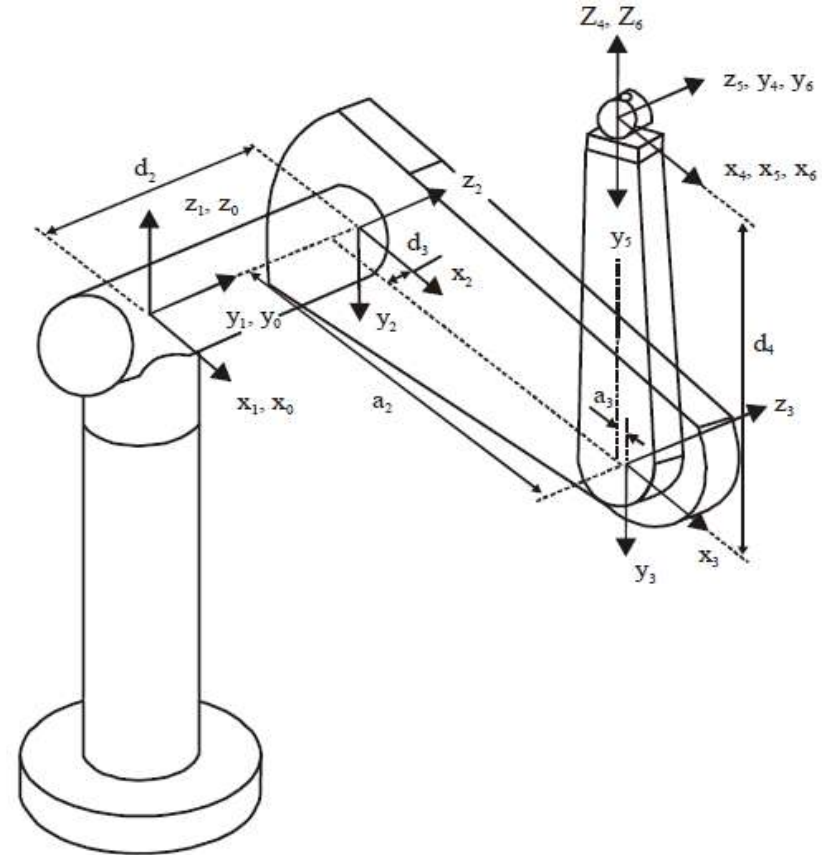


Fig. 1: PUMA 560 in the zero position with attached coordinates frames<sup>17</sup>

# It may get complex very quickly...

$${}^0T_6 = {}^0T_1 {}^1T_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6),$$

$$r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6)],$$

$$r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6,$$

$$r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6),$$

$$r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6),$$

$$r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6,$$

$$r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5,$$

$$r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5,$$

$$r_{33} = s_{23}c_4s_5 - c_{23}c_5,$$

$$p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1,$$

$$p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1,$$

$$p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}.$$

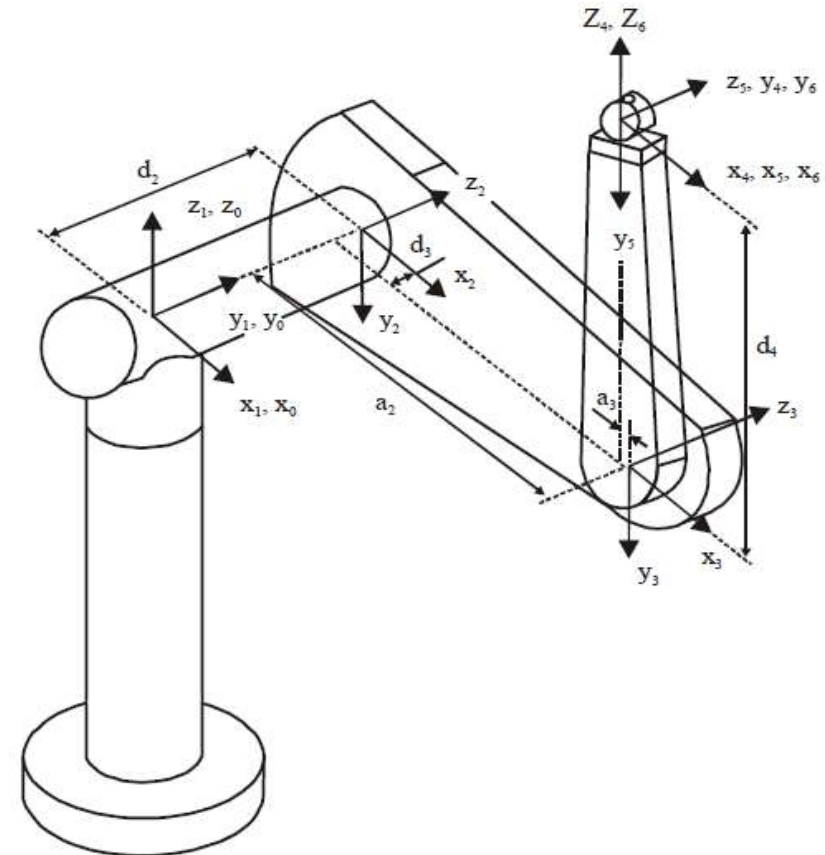


Fig. 1: PUMA 560 in the zero position with attached coordinates frames<sup>17</sup>

# Inverse Kinematics

**Goal:** calculate the joint angles for a desired pose

$$r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6),$$

$$r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6)],$$

$$r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6,$$

$$r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6),$$

$$r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6),$$

$$r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6,$$

$$r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5,$$

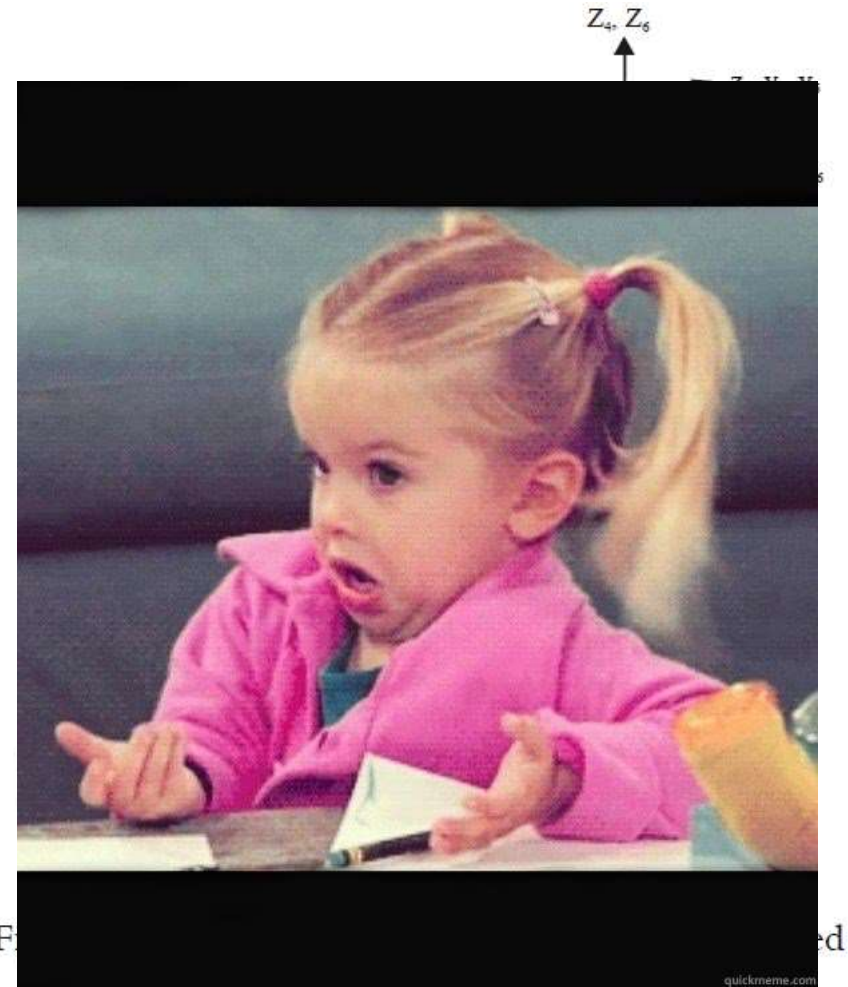
$$r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5,$$

$$r_{33} = s_{23}c_4s_5 - c_{23}c_5,$$

$$p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1,$$

$$p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1,$$

$$p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}.$$



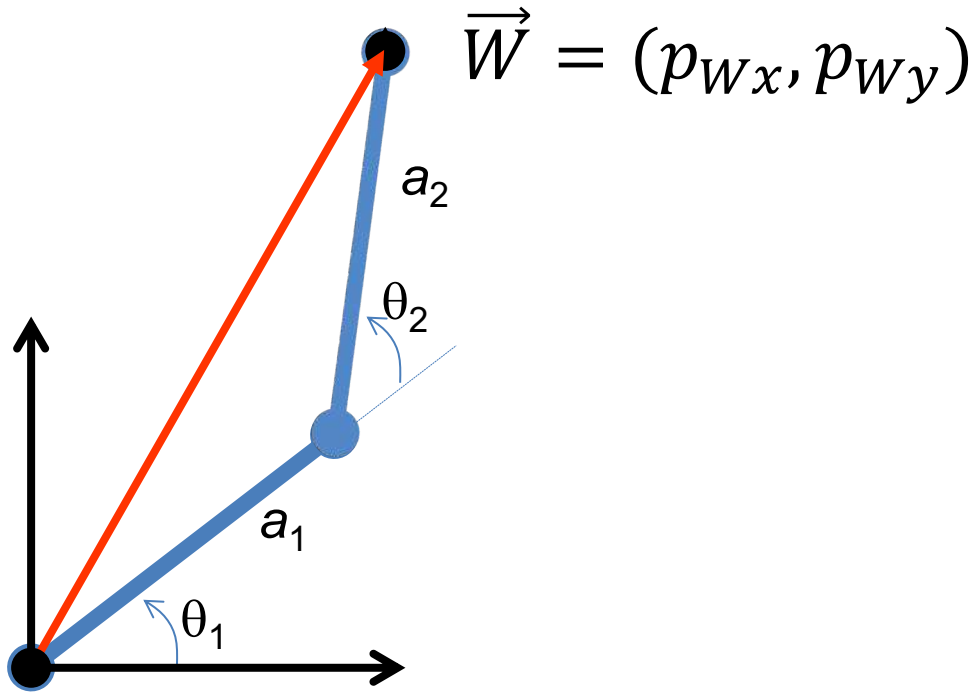
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COORDINATES FRAMES SHOWN

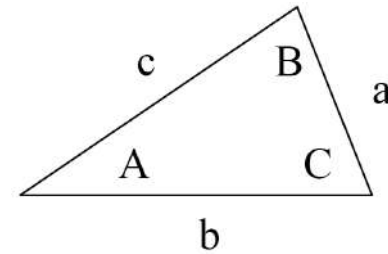
quickmeme.com

# A geometric example



$$\theta_1 = ? \quad \theta_2 = ?$$

## Law of Cosines



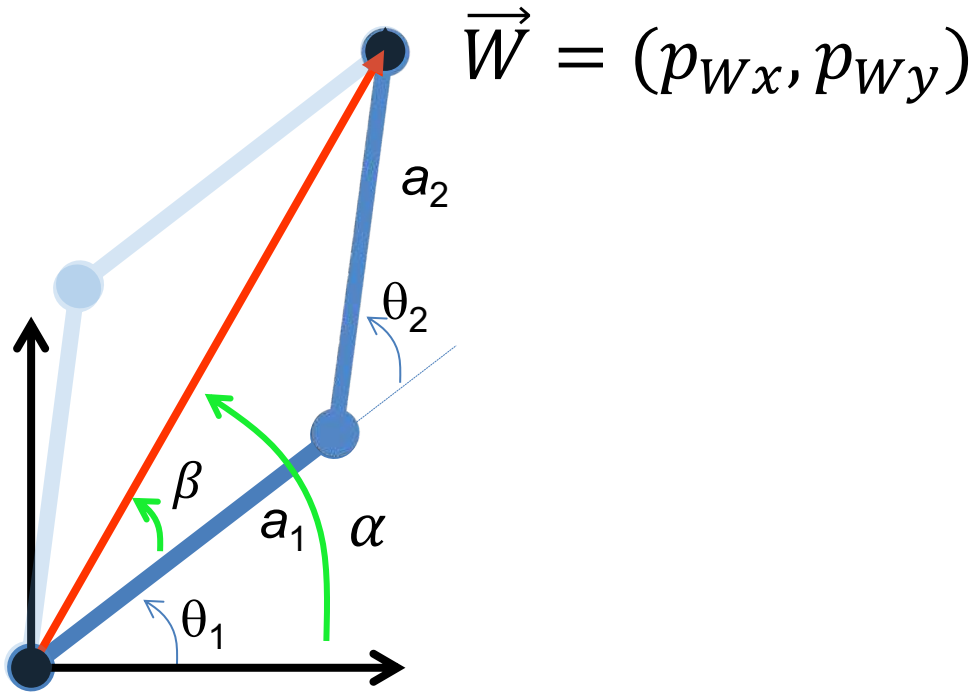
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(\pi - \theta_2) = \frac{a_1^2 + a_2^2 - (p_{Wx}^2 + p_{Wy}^2)}{2a_1a_2}$$

$$\cos(\theta_2) = -\frac{a_1^2 + a_2^2 - (p_{Wx}^2 + p_{Wy}^2)}{2a_1a_2}$$

# A geometric example



$$\theta_1 = ? \quad \theta_2 = ?$$

$$\theta_1 = \alpha \pm \beta$$

$$\alpha = \tan^{-1} \left( \frac{p_{Wy}}{p_{Wx}} \right)$$

$$\beta = \cos^{-1} \left( \frac{a_1^2 + p_x^2 + p_y^2 - a_2^2}{2a_1 \sqrt{p_{Wx}^2 + p_{Wy}^2}} \right)$$

$$\cos(\theta_2) = -\frac{a_1^2 + a_2^2 - (p_{Wx}^2 + p_{Wy}^2)}{2a_1 a_2}$$

# Inverse Kinematics

Even if we find an analytic inverse, the problem may still have zero/multiple/infinite solutions!

$$r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6),$$

$$r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6)],$$

$$r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6,$$

$$r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6),$$

$$r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6),$$

$$r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6,$$

$$r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5,$$

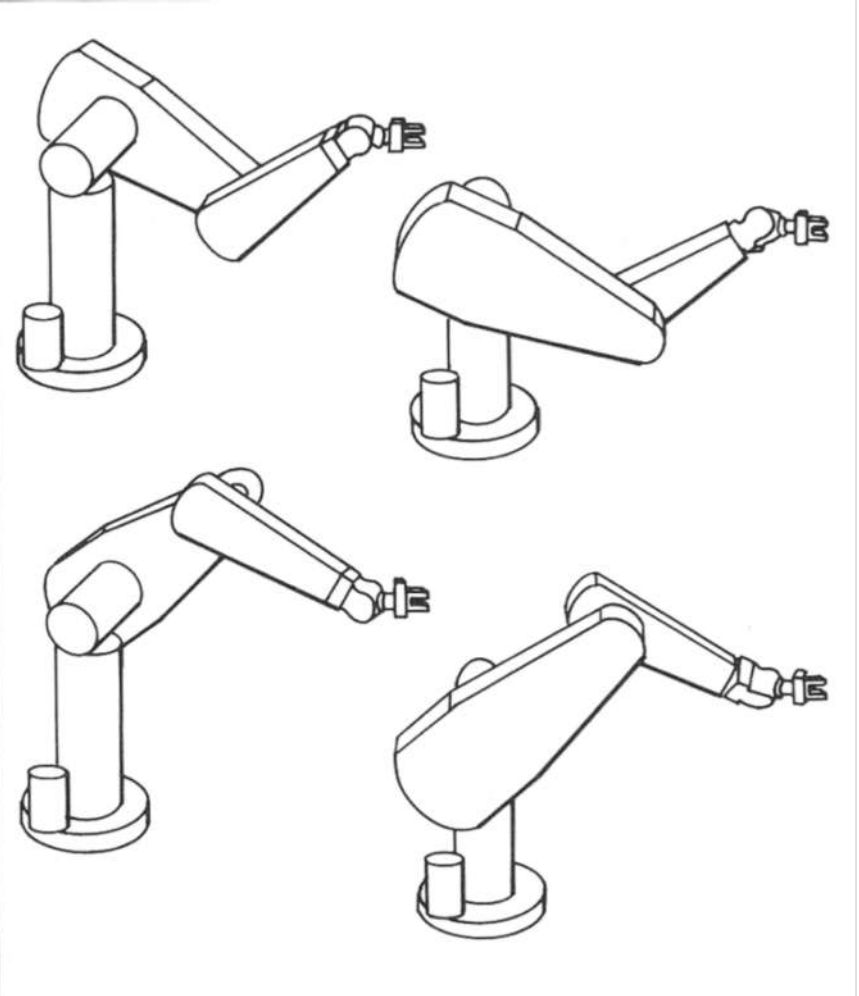
$$r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5,$$

$$r_{33} = s_{23}c_4s_5 - c_{23}c_5,$$

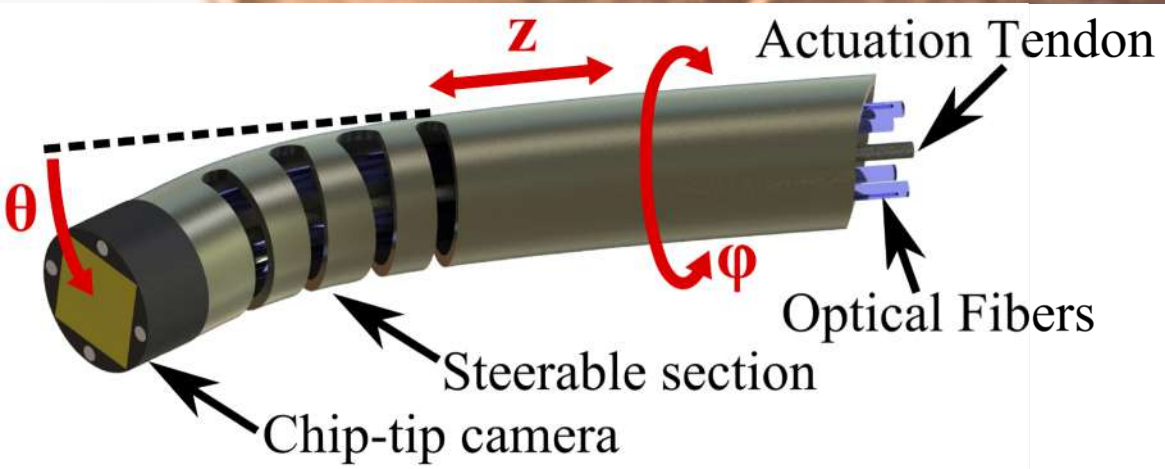
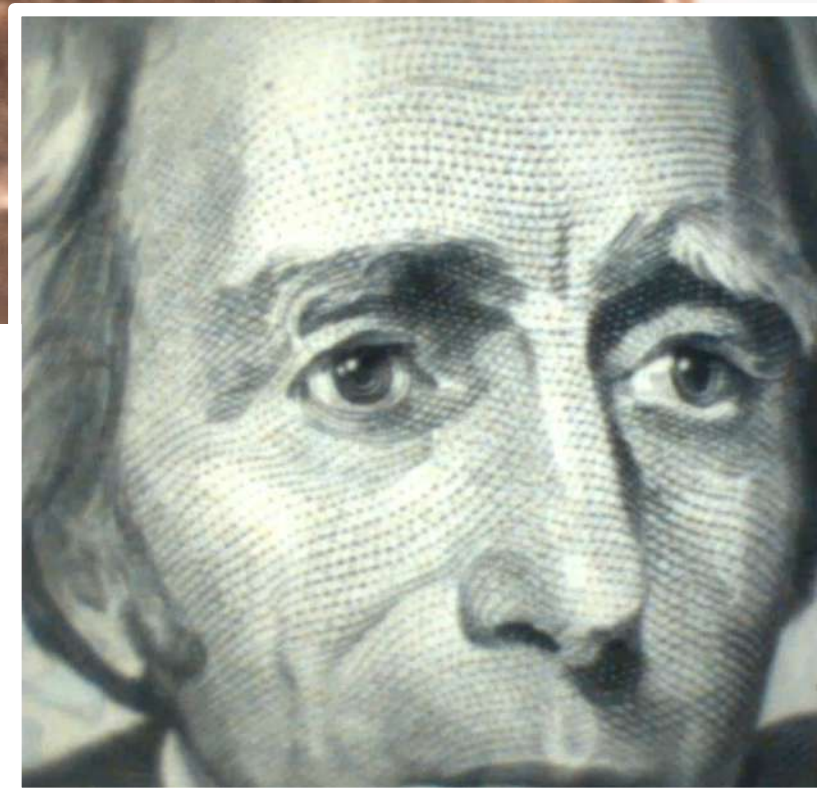
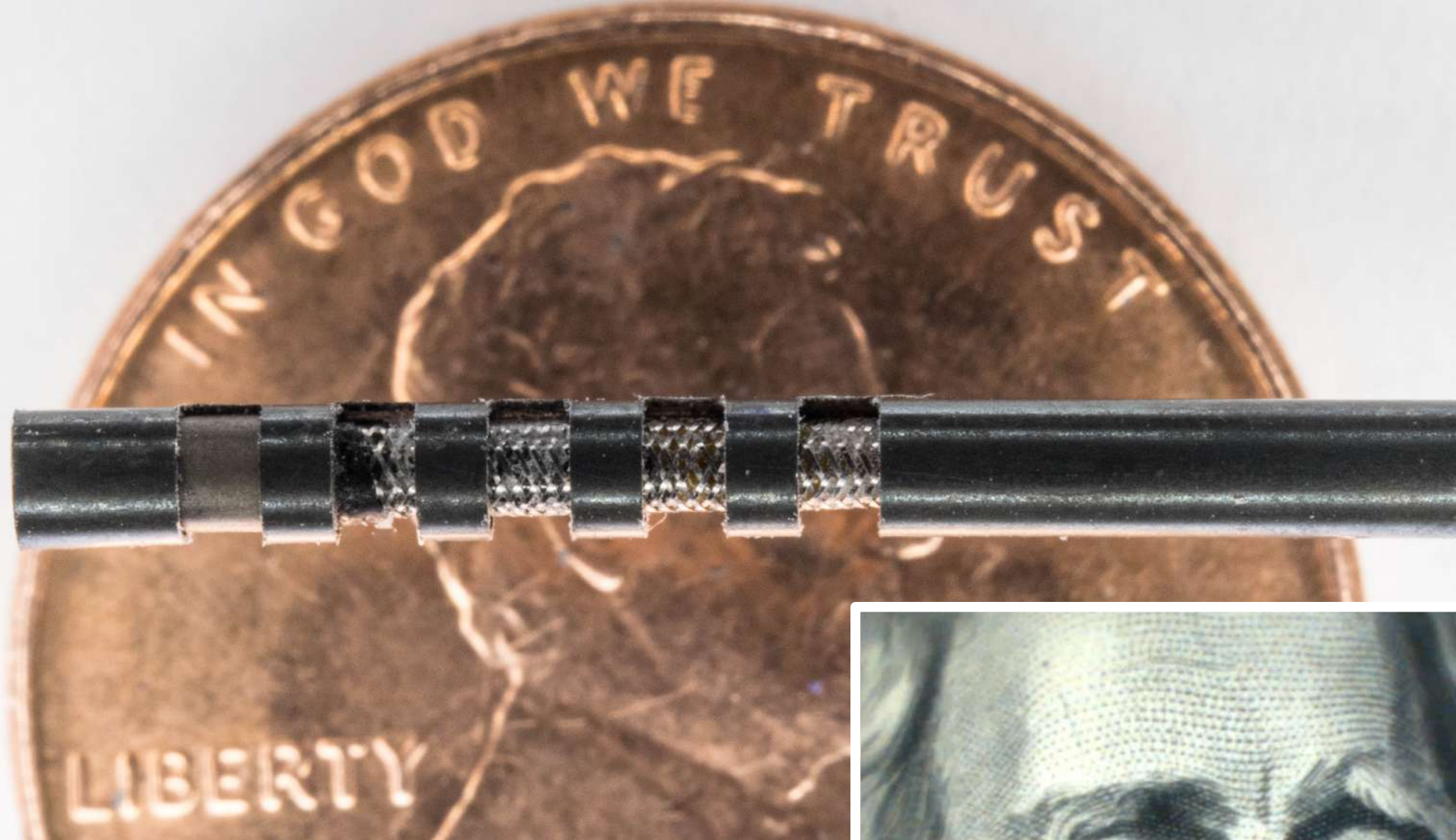
$$p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1,$$

$$p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1,$$

$$p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}.$$

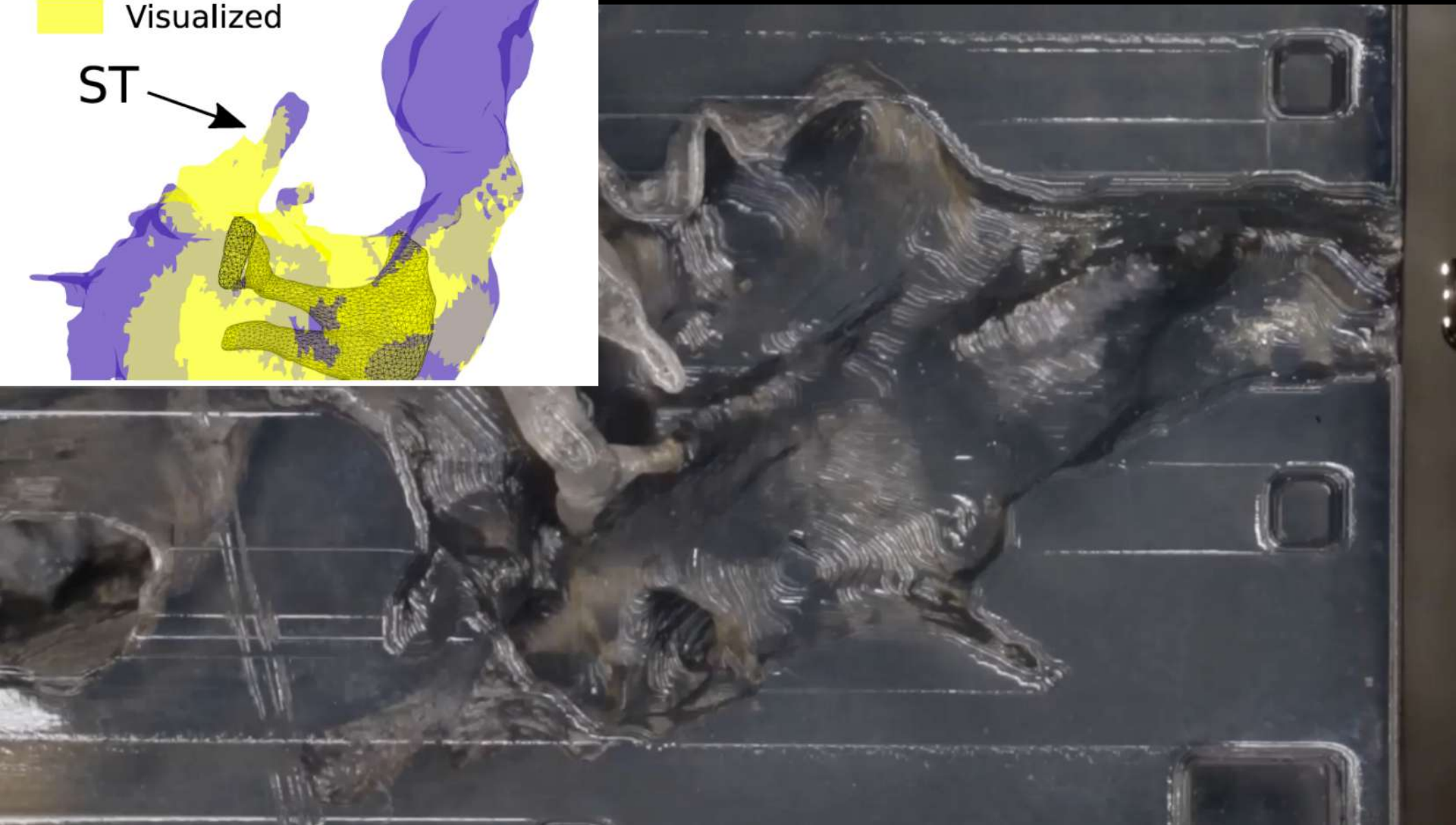
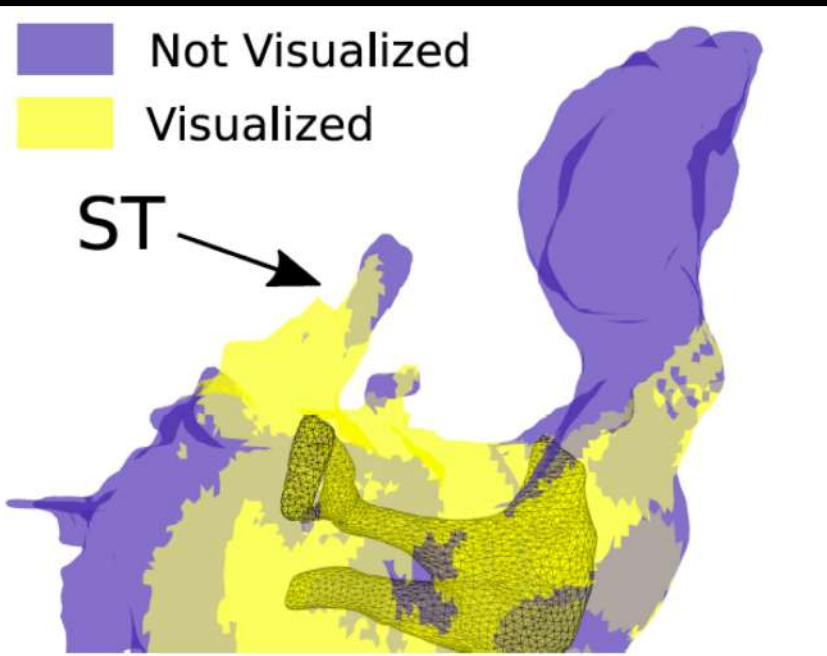


[https://www.youtube.com/watch?v=\\_c6F7mJpSRI](https://www.youtube.com/watch?v=_c6F7mJpSRI)



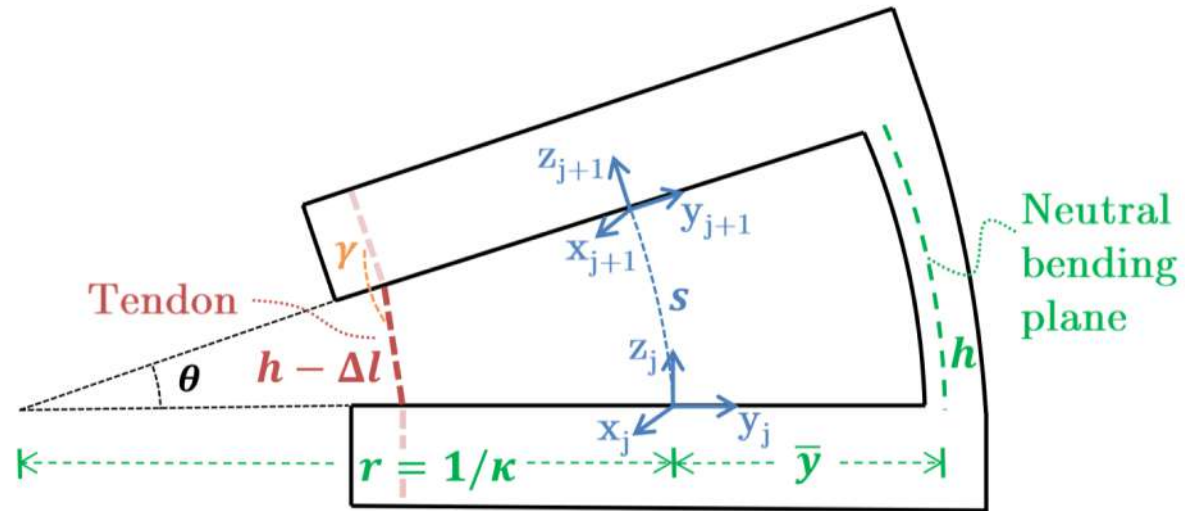
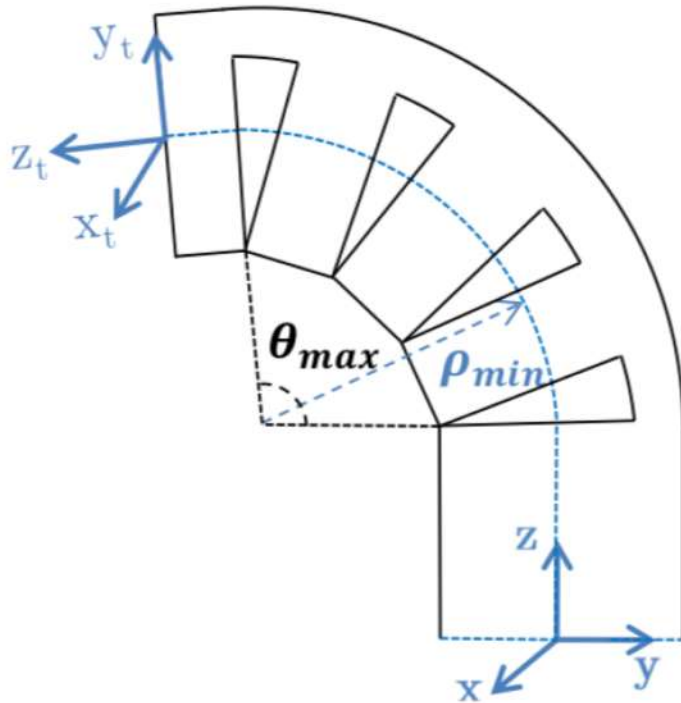
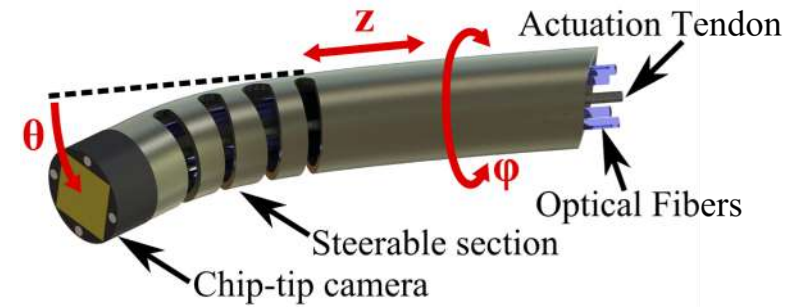


<https://www.youtube.com/watch?v=0nZ7vTxpLQU>



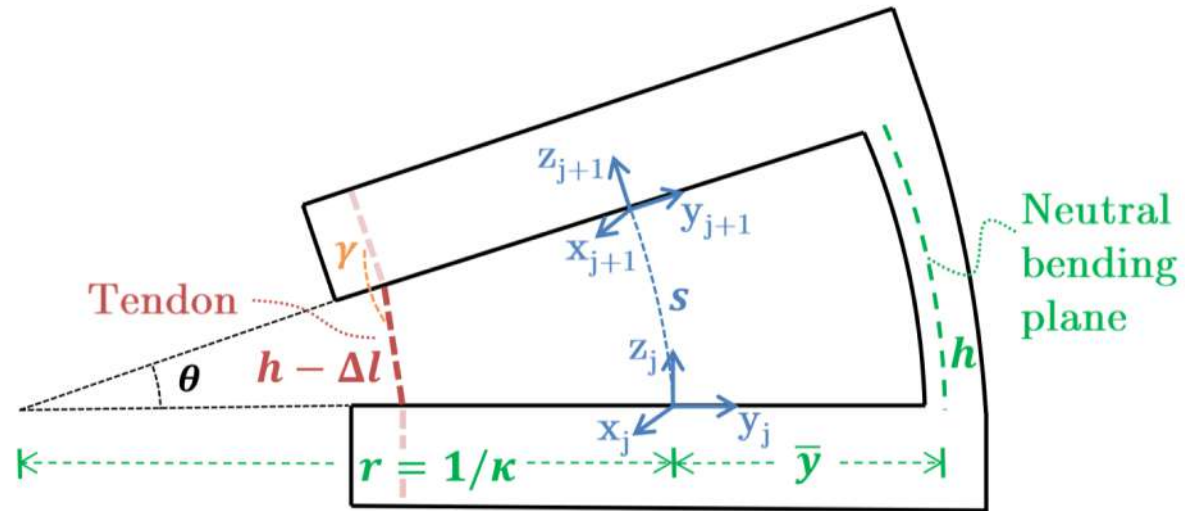
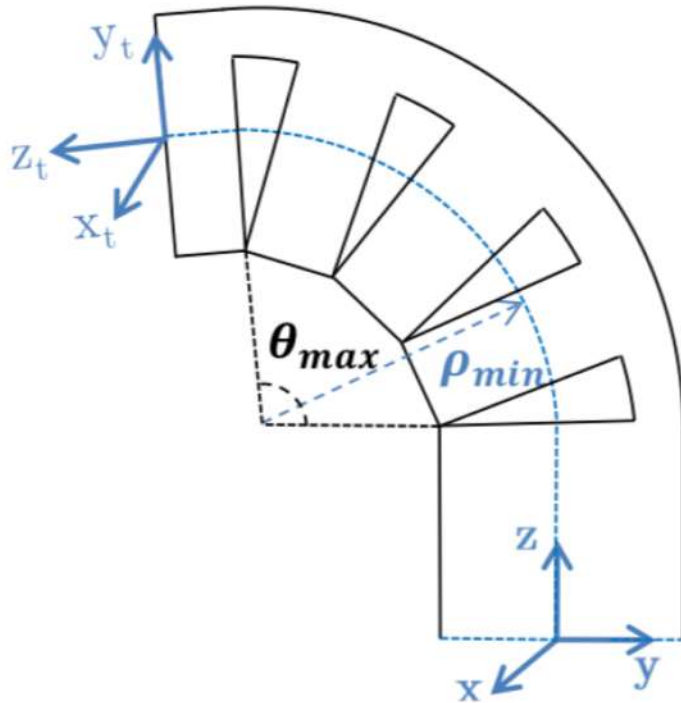
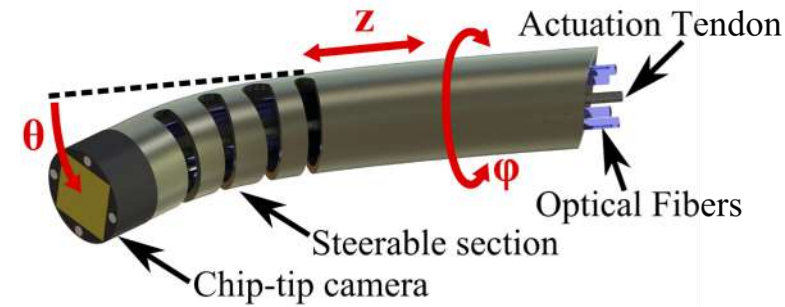
Fichera L et al, **Through the Eustachian Tube and Beyond: A New Miniature Robotic Endoscope to See Into the Middle Ear**, Robotics and Automation Letters (2017)

# Endoscope kinematics



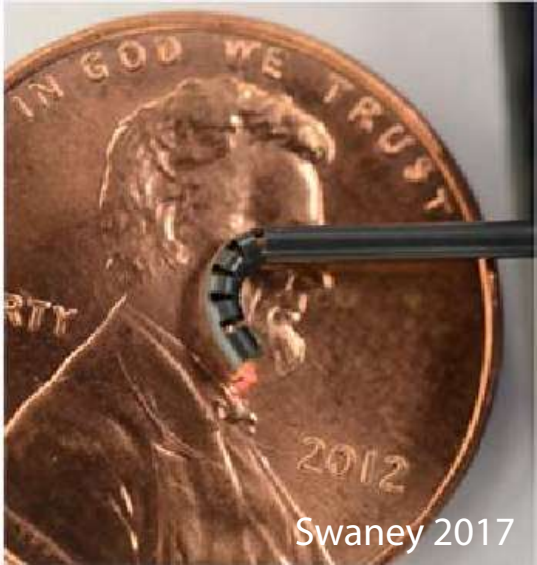
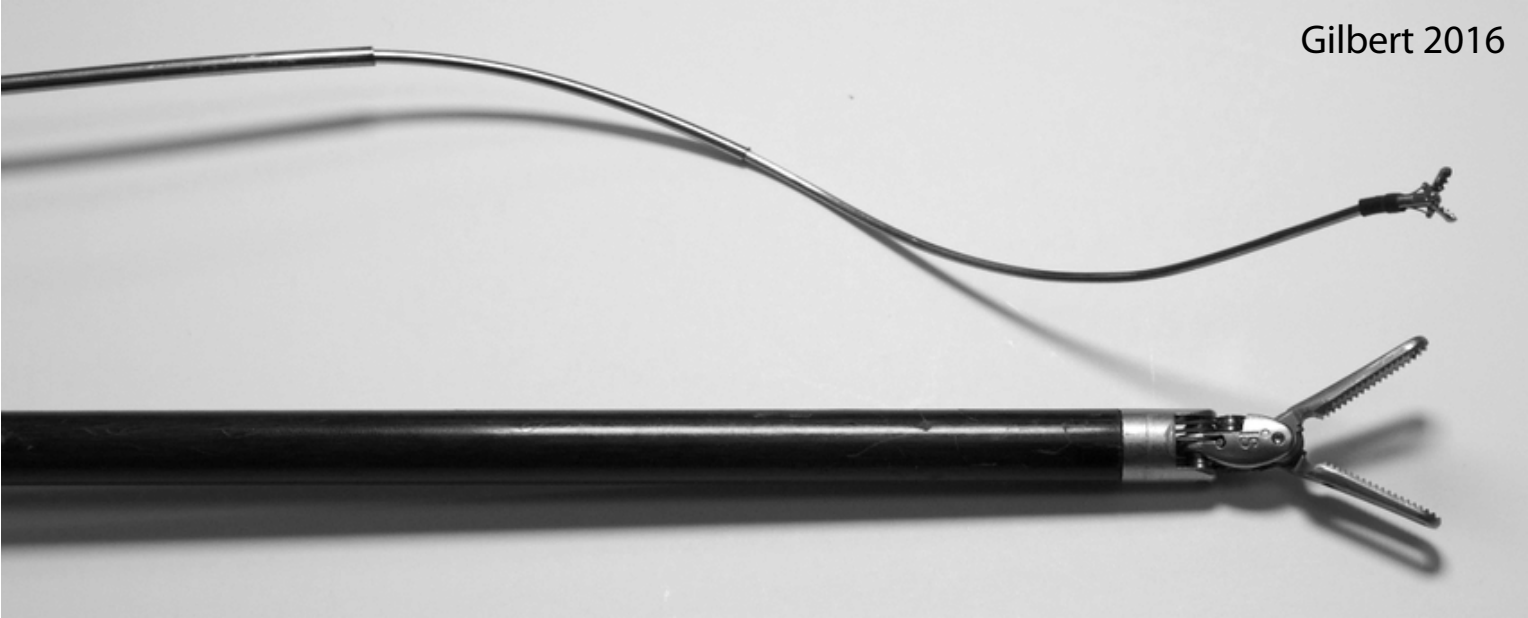
$$\kappa \approx \frac{\Delta l}{h(r_i + \bar{y}) - \Delta l \bar{y}} \quad s = \frac{h}{1 + \bar{y}\kappa}$$

# Endoscope kinematics



$$T_j^{j+1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\kappa s) & -\sin(\kappa s) & (\cos(\kappa s) - 1)/\kappa \\ 0 & \sin(\kappa s) & \cos(\kappa s) & \sin(\kappa s)/\kappa \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Continuum robots in surgery





The Rise of Robots in the Operating Room | Dr. Robert Webster III |  
TEDxNashvilleSalon

<https://www.youtube.com/watch?v=Mr4xEH11N5A>

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