

GCE Syllabus - Linear Algebra

Related course: MA502 - Linear Algebra.

Reference textbooks: Any one of the following.

Linear algebra done right by Axler. Chapters 1-9.

Advanced Linear Algebra by Roman. Chapters 1-3, 7-8.

Linear Algebra and its Applications by Lax. Chapters 1-8.

Expected skills:

- Understanding the foundation of linear algebra: basic concepts, theorems, their connections as well as the fundamental ideas behind definitions and theorems.
- Thinking, arguing, and deducing; proving classic theorems and propositions, and simple modifications.
- Reasoning, calculations and applying concepts and theorems to the solution of open ended problems.

Topics: The questions of the exam will be related to some of the following topics:

1. **Vector spaces:** Axioms and derivations from the axioms; Subspaces, the subspace test, Span; Linear dependence, linear independence, bases; Replacement theorem, dimension of a vector space; Solutions of linear systems.
2. **Linear transformations and matrices:** Definition, elementary properties; Nullspace, Range, the Rank-Nullity Theorem; Ordered bases, the matrix of a linear transformation; Composition of linear transformations, matrix algebra; $\text{Hom}(V, W)$ and its properties; Dual spaces and the dual of a linear transformation; Quotient spaces, the First Isomorphism Theorem; Subspaces invariant under a linear transformation.
3. **Determinants, eigenvalues and eigenvectors:** Eigenvectors and eigenvalues, existence and construction; Determinants and characteristic polynomials; Chains of invariant subspaces, upper triangular forms; Diagonalization; Generalized eigenvectors; The minimal polynomial, Cayley-Hamilton theorem, Jordan Canonical Form; Matrix exponential and applications.
4. **Inner product spaces:** Classification of inner products over \mathbb{R}, \mathbb{C} ; Orthonormal bases, orthogonal complements; Applications, minimisation; Adjoints; self-adjoint, orthogonal and unitary matrices; The spectral theorem; Frobenius norm; Positive operators and polar decompositions; QR decomposition and Least Squares; Singular Value Decomposition.

Exam setting: 3 hours, closed book.

Questions: Usually 6 questions (each may include 2 or more parts).

Note on grading: In order to pass this exam, you must demonstrate sufficient understanding of the underlying definitions, concepts, and methods. Please keep in mind that this is possible without completely solving the problems, but serious mistakes are considered strong evidence of insufficient understanding.

This syllabus has been adopted by the Department of Mathematical Sciences on 12/08/2020 and is overseen by the Graduate Program Committee.