GCE Syllabus - Mathematical Statistics

Related course: MA541 - Probability and Mathematical Statistics II.

Reference textbook:
Statistical Inference (2nd Edition) by Casella and Berger. Chapters 6-10.

Expected skills:

- Understanding the foundation of mathematical statistics: basic concepts, theorems, their connections as well as the fundamental ideas behind definitions and theorems.

- Abilities of thinking, arguing, and deducing; proving classic theorems and propositions on mathematical statistics.

- Abilities of reasoning and applying theorems and calculations to address mathematical statistics related problems connecting to reality.

Topics: The questions of the exam will be related to some of the following topics:

1. Foundational concepts: (a) Sampling from a normal distribution and a uniform distribution; (b) Location-scale families and exponential families; (c) Order statistics, asymptotics.

2. Principles of Data Reduction: (a) Sufficient statistics, factorization theorem, minimal sufficiency, ancillary statistic, sufficiency principle; (b) Completeness, Basu's theorem; (c) Likelihood function, likelihood principle; (d) Equivariance principle.

3. Point Estimation: (a) Methods of estimation: method of moments, maximum likelihood estimation (MLE), Bayes estimators, EM algorithm; (b) Evaluation of estimators: mean-squared error, best unbiased estimators, sufficiency and unbiasedness, loss function optimality.

4. Hypothesis Testing: (a) Methods for test construction: likelihood ratio tests, Bayesian tests, union-intersection and intersection-union tests; (b) Evaluation of tests: error probabilities, power function, p-Values; (c) Most powerful tests.

5. Interval Estimation: (a) Construction of intervals: inversion of tests, pivotal quantities, Bayesian intervals; (b) Evaluation of intervals: size and coverage probability, test-related optimality, maximum likelihood intervals.

6. Asymptotic Evaluations: (a) Point estimation: consistency and efficiency; (b) Robustness; (c) Hypothesis testing: asymptotic distribution of LRTs; (d) Interval estimation.

Exam setting: 3 hours, closed book.

Questions: Usually 6 questions (each may include 2 or more parts).

Note on grading: In order to pass this exam, you must demonstrate sufficient understanding of the underlying definitions, concepts, and methods. Please keep in mind that this is possible without completely solving the problems, but serious mistakes are considered strong evidence of insufficient understanding.

This syllabus has been adopted by the Department of Mathematical Sciences on 12/08/2020 and is overseen by the Graduate Program Committee.