Three Resources

Suzanne L. Weekes

Department of Mathematical Sciences Worcester Polytechnic Institute



Three Questions

1. What is mathematics?

2. Why do you think that students need to learn mathematics?

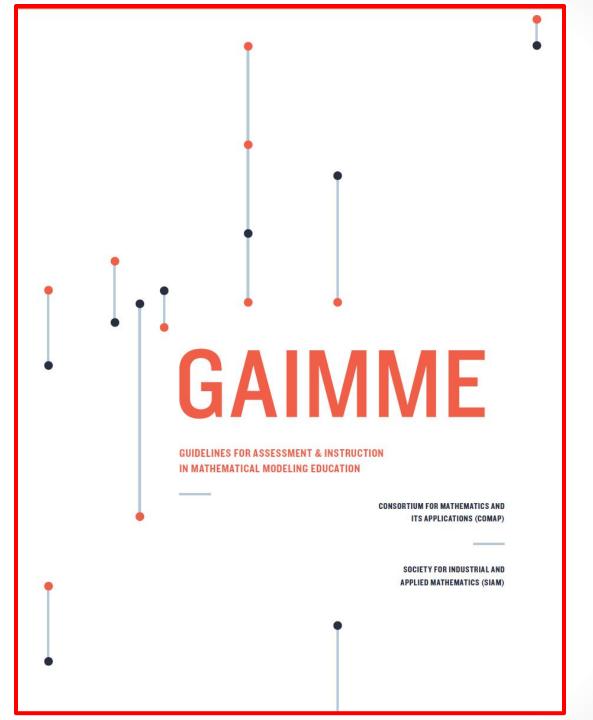
3. What are the important things that they need to know?

Three Resources

1. GAIMME Report – COMAP & SIAM

2. Industrial Math Projects for High School Students (IMPHSS) - CIMS

3. PIC Math videos







PREFACE

INTRODUCTION

In 2015, leaders from SIAM and COMAP came together to produce a report we have named GAIMME — Guidelines for Assessment and Instruction in Mathematical Modeling Education. The name gives homage to the fine work of the American Statistical Association's impressive GAISE report. Like that document our primary audience is you the teacher. While we hope that test and policy makers will read this document and use it in their decision-making, it has been written for the front-line teacher. We hope and intend that these guidelines will be of help as you incorporate the practice of mathematical modeling into your classrooms.

A major reason for the creation of GAIMME was the fact that, despite the usefulness and value in demonstrating how mathematics can help analyze and guide decision making for real world messy problems, many people have limited experience with math modeling. We wanted to paint a clearer picture of mathematical modeling (what it is and what it isn't) as a process and how the teaching of that process can mature as students move through the grade bands, independent of the mathematical knowledge they may bring to bear.

HOW TO USE THIS DOCUMENT

While we very much hope that you have the time and interest to read this document cover to cover, it has been written to permit a more focused reading as well. The opening section, "What is Mathematical Modeling?" is intended for every reader and contains the main arguments for the inclusion of mathematical modeling across all grade levels. It is a

Guidelines for Assessment and Instruction in Mathematical Modeling - GAIMME

"Despite the usefulness and value in demonstrating how mathematics can help analyze and guide decision making for real world messy problems, many people have limited experience with math modeling."

Goal of GAIMME Report

Clearer picture of mathematical modeling

- ✓ what it is and what it isn't as a process
- ✓ how the teaching of that process can mature as students move through the grade bands

CONTENTS

WRITING TEAM:

What is Mathematical Modeling?: Karen Bliss*, Jessica Libertini

 $\textit{Early and middle grades:} \ \textbf{Rachel Levy}^*, \textbf{Rose Mary Zbiek}, \textbf{Ben Galluzzo}, \textbf{Mike Long}$

 ${\it High \, school:} \, Dan \, Teague^*, Landy \, Godbold, Joe \, Malkevitch, Henk \, van \, der \, Kooij$

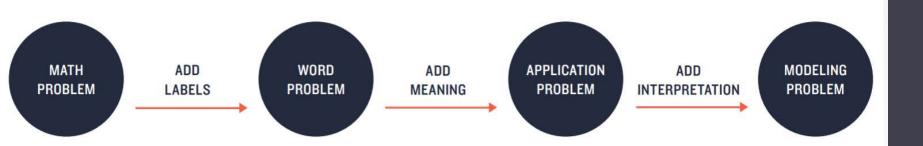
 ${\it Undergraduate:} \ {\it Frank Giordano}^*, Karen \ Bliss, Katie \ Fowler, Henry \ Pollak, Jessica \ Libertini$

Resources: Heather Gould

Editors: Sol Garfunkel*, Michelle Montgomery

PREFACE	iii	
CHAPTER I WHAT IS MATHEMATICAL MODELING?	7	
CHAPTER 2 MATHEMATICAL MODELING IN THE EARLY AND MIDDLE GRADES:		
PREKINDERGARTEN THROUGH GRADE 8	23	
CHAPTER 3 MATHEMATICAL MODELING IN HIGH SCHOOL: GRADES 9 THROUGH 12	45	
CHAPTER 4 MATHEMATICAL MODELING AT THE UNDERGRADUATE LEVEL	71	
CHAPTER 5 WHAT IS MATHEMATICAL MODELING, THE ART AND FLAVOR	95	
APPENDIX A MATHEMATICAL MODELING RESOURCES	103	
APPENDIX B MODELING EXAMPLES, ELEMENTARY AND MIDDLE GRADES	119	
APPENDIX C EXTENDED EXAMPLES	147	
APPENDIX D ASSESSMENT TOOLS	193	
REFERENCES	207	

Math Problem --> Modeling Problem



 $300 \div 25 =$

 $295 \div 25 =$

Our school has 300 students. A bus holds 25 students. How many buses are needed to take the students on a field trip? Our school has 295 students. A bus holds 25 students. How many buses are needed to take the students on a field trip?

Our school has 295 students. What is the best way to transport the students for the field trip?

Karen Bliss, Katie Kavanaugh, Ben Galluzzo



Altering Students' Belief

Replace

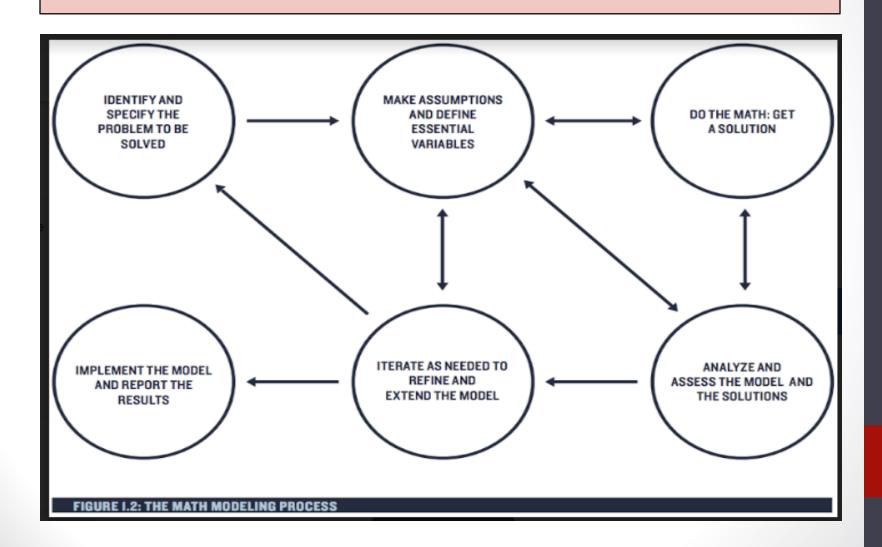
"Math is rigid. It requires remembering how."

with

"Math is open and empowering.

It requires creativity and figuring out how."

The GAIMME Modeling Cycle



Modeling Guiding Principles

- Modeling is open ended and messy
- When students are modeling, they must be making genuine choices
- Modeling problems can be developed from familiar tasks
- Assessment should focus on the process and not on the product or pieces
- Modeling happens in teams.

Modeling Classes are Difficult

Discuss with Parents

Discuss with Faculty Colleagues

Discuss with Administration

Sample Modeling Question

Every driver recognizes the fluctuations in gas prices that happen almost on a weekly basis.

Phone apps can map stations' prices and locations, and in some areas, a local radio station has a special report on the location of the gas station with the lowest price per gallon for regular gas. Of course, that station is likely to be across town from where you are driving.

If you know the locations and the prices at several gasoline stations, at which station should you buy your gas? Does it matter if you think that you are buying gallons of gas or that you are buying miles of travel? Develop a model that can be used by drivers of different cars that will tell them how far they should be willing to drive based on the specifications of their car.

Turn in four PowerPoint slides, one with the assumptions, one with the mathematical model, and two showing the screens for an app based on your model. The first app screen should request essential information from the user, and the second should show the app's response.

Sample Modeling Question

A mantid is a small, crawling insect that closely resembles a cockroach. Mantids are often used in biological studies because they move very slowly, so it is easy to keep track of them. Mantids move primarily to seek food. Researchers have been studying the relationship between the distance a mantid will move for food and the amount of food already in the mantid's stomach. The distance is measured in millimeters and the amount of food in centigrams (a hundredth of a gram). In the research, food was placed progressively nearer to a mantid and the distance at which the mantid first began to move towards the food was noted. The amount of food in the mantid's stomach was also measured. Measurements for 15 mantids are given below:

FOOD (CG)	11	18	23	31	35	40	46	53	59	66	70	72	75	86	90
DISTANCE (MM)	65	52	44	42	34	23	23	8	4	0	0	0	0	0	0

Based on the data provided, determine a functional relationship between the amount of food in the mantid's stomach and the distance it will walk to eat. Biologists call the amount of food in the stomach at which an animal will begin to seek food the hunger threshold. Based on your functions, what is the hunger threshold for the mantid?

Center for Industrial Mathematics & Statistics



- Founded in 1997
- Make partnerships with industry that benefit the sponsors, and our mathematical sciences community
- Real-world research projects that come directly from industry, government and finance

Visit http://www.wpi.edu/+CIMS

CIMS Industrial Partners

130+ projects

75+ companies















SRI International













Industrial Math Projects for High School Students

http://labs.wpi.edu/imphss/

- 20+ industrial mathematics projects for high school students drawn from a variety of real-world situations.
- Projects for every level of high school mathematics, from Algebra to Calculus and Statistics
- Flexible length and scope of the projects
- Project database contains downloadable versions of each of the projects, ready to be assigned to students.









Criminal Intentions



Child Mortality



Ceramic Powder Manufacturing



Ceramic Capacitors



BioMass



You're in the Driver's Seat



Wearing Through the Pipe



Vapor Recovery Systems



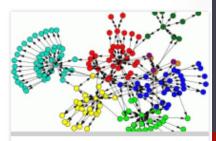
Taffy Production Line



Super Soakers



Six Sigma



Network Damage



Network Analysis



Mining the MCAS- Statistics



Listen to Periodic Functions



Investment Banking

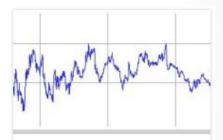


Moral Hazard Test

ISD Certification Testing



The Illusion of Control



The Geometry of Investing



Enter the iBot



The Enigma Key

NCTM Standards

Click here for a copy of the NCTM Standards for Mathematics

	Suggested Classes	Number and Operations	Algebra	Geometry	Measurements	Data Analysis and Probability
BioMass Research and assess alternative energy sources	Modeling	12.N.9	12.A.6 12.A.14		12.M.1 12.M.5	
Ceramic Capacitors Maximize yield and profit in a manufacturing process	Pre-Alg Algebra 1 Geometry	8.N.1 8.N.4 8.N.7 8.N.11 8.N.14	8.A.2 8.A.4 8.A.6 8.A.7 12.A.9 12.A.12 12.A.14		8.M.2 8.M.3 8.M.8 8.M.9 12.M.1	
Ceramic Powder Manufacturing Determine parameters for powder manufacturing	Pre-Alg Algebra 1 Geometry	8.N.1 8.N.4 8.N.7 8.N.11 8.N.14	8.A.2 8.A.4 8.A.6 8.A.7 12.A.9 12.A.12 12.A.14		8.M.2 8.M.3 8.M.8 8.M.9 12.M.1	
Child Mortality Study factors that effect life insurance cost	Statistics Modeling	12.N.9	12.A.2 12.A.5 12.A.6 12.A.9 12.A.11 12.A.12 12.A.13		12.M.1	12.D.5
Criminal Intentions Determine where in the 50 states it's safest to live	General Algebra Statistics					8.D.2 8.D.3 8.D.4 12.D.4 12.D.5
DMAIC - Algebra Analyze a company and recommend areas to improve	Version 1 Pre-Al General Algebra	8.N.1 8.N.11				8.D.2 8.D.3 8.D.4 12.D.4 12.D.5
DMAIC - Statistics	Version 2					12.D.4 12.D.5

Industrial Mathematics Project for High School Students - Projects

<u>Home</u> | <u>Projects</u> | <u>Standards</u> | <u>Tutorials</u> | <u>Sponsors</u> | <u>About</u> | <u>Contact</u>

Click a project name for more information. Each project has an explanation and a classroom-ready downloadable version.

Sort Projects: Alphabetically By Grade Level By Industry

Grade Levels: General Math Geometry Statistics
Pre-Algebra Algebra II Calculus
Algebra Pre-Calculus Modeling







Sample Projects

Taffy Production Line



Download this project

The Problem

Modern production lines have to be designed to be as fast as possible with almost no error. When producing millions of products a day how does a company keep track of it all? The project describes how salt water taffy is made. The process starts with mixing raw materials, which are then baked in ovens and subsequently cooled. The taffy ingredients are pulled, folded and twisted on special pulling machines. Finally, the mixture is put into an extruding and cutting machine which produces pieces of wrapped taffy. In this project students become design engineers. They will need to decide how many of each machine the company needs, how long it will take to make taffy, and how much raw material is needed each day.

Network Analysis



Download this project

The Problem

Amtrak has been shown to be a company that does a service to the country, but does not make a profit. Executives at Amtrak have suggested that some of the rail lines, stations and/or trains should be eliminated to allow the company to minimize its losses. A team of consultants has been hired to analyze the rail services that are available in any region and decide where Amtrak needs to concentrate its money and services to best meet the needs of its customers. As a team of consultants you will need to make recommendations in your area of expertise as to what Amtrak should do.

Areas of Application

PICMath

Preparation for Industrial Careers in Mathematical Sciences



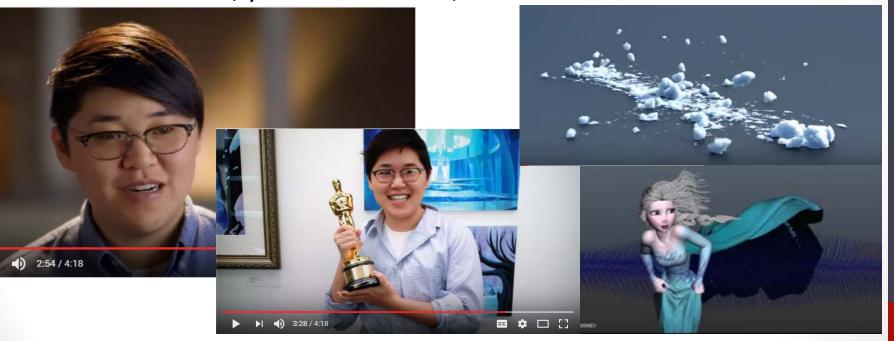




Watch videos at http://picmath.com

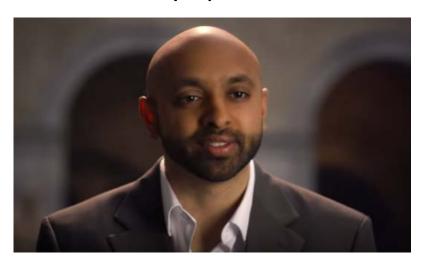
Creating Realistic Animations

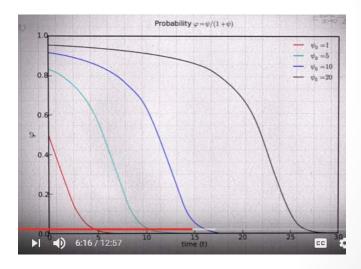
Dr. Alex McAdams, Senior Software Engineer at Walt Disney Animation Studios, talks about how mathematics is used to make realistic, yet art-directable, animations.

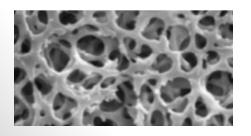


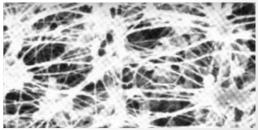
Building a Better Filter

Dr. Sumanth Swaminathan of W. L. Gore & Associates talks about some research questions about filtration. He works to understand the different waste capture mechanisms of filtration devices and to mathematically optimize the microstructure to create better filters.









Finding the Safest Place to Store Nuclear Waste

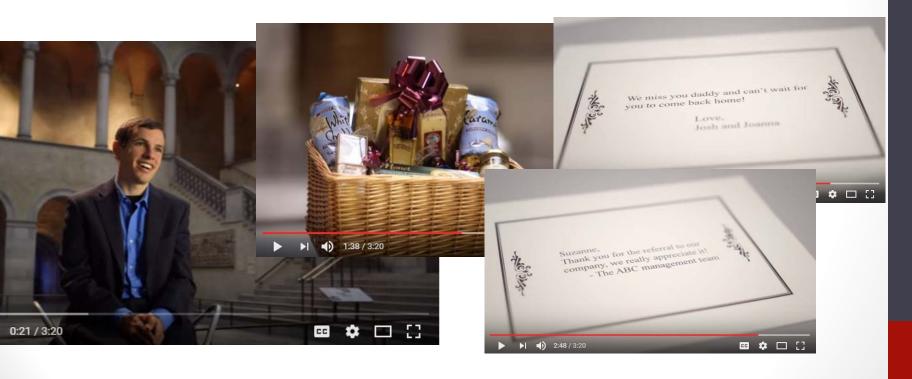
Dr. Genetha Gray talks about a Sandia National Labs problem in which they used limited geological data to create a groundwater flow model, with parameters to be determined, to study the feasibility and safety of prospective subsurface nuclear waste storage sites.





Improving Market Strategies

Dr. Jonathan Adler Nolis helped an online gift basket company distinguish between its business customers and its private consumers from the gift messages.



Discussion