Mathematical Modeling of Torque for Screw Insertion Processes

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2005-2006 Major Qualifying Project

Department of Mathematical Sciences

The 2006 Provost MQP Honorable Mention

The 2006 CIMS MQP Award

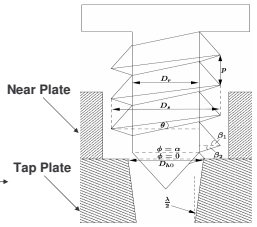
Abstract –

A self-tapping screw is a high-strength one-piece fastener that is driven into preformed holes. We analyze and improve a mathematical model [1] of the self-tapping screw insertion process so that it can be used in manufacturing processes at the Bose Corporation. We build a Graphical User Interface in MATLAB which allows users to enter fundamental data and produces the corresponding torque curve. The accuracy and robustness of the model is tested by comparing predictions to empirical data collected at Bose.

1. Background

- The torque applied to the screw must be in a certain range in order for proper fastening. The industry needs a way to predict the correct range of torque values in order to decrease reliance on experimentation.

Screw-Hole System



This figure illustrates the key parameters used in the model.

Model Assumptions

- Heat transfer effects during screw fastening are negligible.
- The thread cross-sectional area changes uniformly throughout the tapered part.
- The driver applies sufficient axial force and torque to allow the screw to advance.
- The thread helix angle, θ, is constant along the entire screw.

2. Mathematical Model

- Use helical coordinate variable ϕ to mark the angular position on the screw thread.
  - $\phi = 0$ represents first instant of contact between screw and hole.
  - $\phi = \alpha$ represents first point on the untapered portion of screw.
- Identify all forces acting on an area δA of the screw.
  - The resulting elemental force vector depends on stress and area.
  - For example, $\delta\vec{F}_s = \vec{f}(\bar{\phi},\phi)\sigma_{uts}\delta A$
- Find expressions for δA in terms of screw and hole parameters.
- Using $\delta\vec{M}_s = \vec{R}_s \times \delta\vec{F}_s$, find the moment created by the forces on the screw.
  - $\vec{R}_s$ is the position vector from the top of the tap hole to the point of action of the forces.
  - Torque counteracts moment: $\delta\vec{T}_s = -\delta\vec{M}_s$
  - Only *z*-component needed.
- Integrate torque equations over region of action.

3. Displacement of Hole Material & Engagement

- Approximate slant hole as piecewise-straight hole and derive expression for elemental force,

$$\delta\vec{F}_s = \frac{A_c\sigma_{uts}}{\alpha - \frac{1}{2}\kappa_{\max}}\vec{f}(\phi)\delta\phi$$

  - Thread cutting area decreases linearly on the taper.
  - Set stress equal to ultimate tensile strength to allow for maximum plastic deformations.
- Need to find distance from screw central axis to centroid of cutting region because force acts at center of mass.
  - Distance to approximate centroid is $\hat{r}_s$
- Final Torque Equation: $T_{sz}(\bar{\phi}) = \int_{\zeta(\bar{\phi})}^{\psi(\bar{\phi})} dT_{sz} \approx \int_{\zeta(\bar{\phi})}^{\psi(\bar{\phi})} \frac{A_c\sigma_{uts}\hat{r}_s\cos(\theta)}{\alpha - \frac{1}{2}\kappa_{\max}}d\phi$

Initial Engagement

- Engagement, which represents the beginning of the screw insertion process, is a combination of displacing material and overcoming friction.
- Major difference is change in displacement torque – only elastic deformation is expected during the initial engagement; so σ_y is used instead of σ_{uts}.
- Final Torque Equation:

$$T_{ez} = T_{dz} + T_{fz} = \frac{A_c\sigma_y\hat{r}_s\cos(\theta)}{\alpha - \frac{1}{2}\kappa_{\max}}\left[\psi(\bar{\phi}) - \zeta(\bar{\phi})\right] + \int_{\zeta(\bar{\phi})}^{\bar{\phi}} K_f(\bar{\phi},\phi)r_f B(\bar{\phi},\phi)d\phi$$

4. Overcoming Friction & Tightening

The expression for elemental force due to friction:

$$\delta\vec{F}_f = K_f(\phi-\phi)\left(\sec(\beta_1 - \tfrac{1}{2}\lambda)\begin{bmatrix}-\sin(\beta_1)\cos(\bar{\phi}-\phi)+\cos(\beta_1)\sin(\bar{\phi}-\phi)\sin(\theta)-\mu\sin(\bar{\phi}-\phi)\cos(\theta)\\-\sin(\beta_1)\sin(\bar{\phi}-\phi)-\cos(\beta_1)\cos(\bar{\phi}-\phi)\sin(\theta)+\mu\cos(\bar{\phi}-\phi)\cos(\theta)\\-\cos(\beta_1)\cos(\theta)+\mu\sin(\theta)\end{bmatrix}\right.$$
$$\left.+\sec(\beta_2+\tfrac{1}{2}\lambda)\begin{bmatrix}-\sin(\beta_2)\cos(\bar{\phi}-\phi)-\cos(\beta_2)\sin(\bar{\phi}-\phi)\sin(\theta)-\mu\sin(\bar{\phi}-\phi)\cos(\theta)\\-\sin(\beta_2)\sin(\bar{\phi}-\phi)+\cos(\beta_2)\cos(\bar{\phi}-\phi)\sin(\theta)+\mu\cos(\bar{\phi}-\phi)\cos(\theta)\\\cos(\beta_2)\cos(\theta)+\mu\sin(\theta)\end{bmatrix}\right)\delta\phi$$

where $K_f(\bar{\phi},\phi) = \frac{1}{2}\left(D_s(\phi) - D_h(\bar{\phi},\varphi)\right)\sigma_f r_f \cos(\frac{\lambda}{2})\sec(\theta)$

Compressive strength, σ_f, is the maximum compressive stress that the material can sustain.

Integrate torque numerically, $T_{fz}(\bar{\phi}) = \int_{\zeta(\bar{\phi})}^{\bar{\phi}} dT_{fz} = \int_{\zeta(\bar{\phi})}^{\bar{\phi}} K_f(\bar{\phi},\phi)r_f B(\bar{\phi},\phi)d\phi$

Tightening

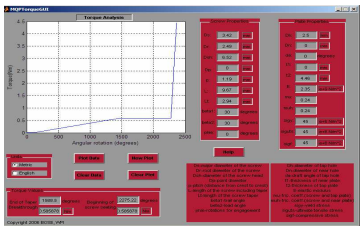
- Screw head encounters friction from surface of near plate – this force is tangential to the direction of screw motion.
- Screw head also experiences compressive forces from near plate – the plate pushes up against the screw.

$$T_{bi} = \int \delta M = \int \mu_h\sigma_z r dA = \int_{D_{sh}/2}^{D_h/2}\int_0^{2\pi}\mu_h\sigma_z r^2 dq\, dr = \tfrac{1}{12}\pi\mu_h\sigma_z\left(D_{sh}^3 - D_h^3\right)$$

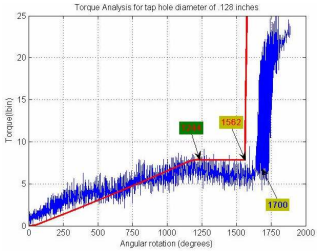
6. Results

Both screws were inserted in ABS and reached full breakthrough. The discrepancy between the model and experimental curves is believed to be due to the presence of heat.

Graphical User Interface

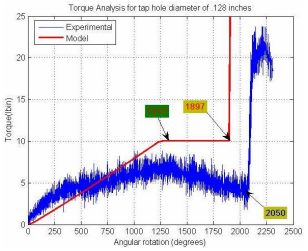


Asymmetric threads with gimlet tip



Green box – end of taper breakthrough

Asymmetric threads with blunt tip



Yellow box – beginning of tightening

5. Conclusions & Future Work

- Friction is most dominant force acting on the screw in the insertion process.
  - The compressive strength of the plate material is an important factor in deciding how much torque it takes to insert a screw.
- The draft angle, because it is usually very small, does not affect the torque profile significantly.
- When the thread area in contact with the hole is increased, the torque required increases as well.
  - The symmetry of the thread influences this area.
- In the future, accounting for the heat transfer effects and when the screw is sufficiently tightened will improve the model's accuracy.
- Given certain screw and plate materials, what is the optimal screw and hole geometry to insure proper fit for the intended purpose?

Reference

[1] Seneviratne, L., Ngemoh, F., Earles, S. and Althoefer, K., Proc. IMechE, Pt. C, J. Mech. Enging Sci., 215, 2001.

Acknowledgments

Special thanks to:

Peter Blatchley and Tom Dawe, Bose Corp.

Advisor: Prof. Suzanne L. Weekes

Sponsor: Bose Corporation