Optimization of the Sierpinski Carpet Fractal Antenna

B r i a n   G.   C o r d e s

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Abstract –

This project studies the Sierpinski Carpet fractal antenna in terms of optimality. A 3D full-wave FDTD model of the structure is developed and used in conjunction with a neural network optimization procedure. An original engineering design based on the idea of a broken fractal geometry is suggested. The modified fractal antennas are shown to have the capability of being optimized, in terms of its return loss, in a narrow frequency band.

Related Fundamental Issues

The self-similarity properties of fractal shapes can be translated into electro-magnetic behavior and may result in a multiband antenna. Yet, studies of fractal antennas have been practically limited to rather *non-systematic explorations of design ideas* and *nearly accidental findings of fortunate characteristics.*

- **Since fractals are known to be suitable models for nature, *is the fractal geometry of an antenna optimal in some sense*?**
- ***Which properties* (if any) of fractal antenna geometries *may be responsible* for multiband/wideband performance?**

1. The Sierpinski Carpet Fractal

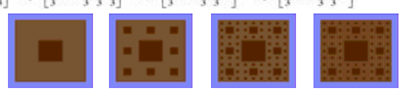
This project is focused on the Sierpinski Carpet (SC) fractal and the related antenna. The SC fractal is defined using eight specific affine transformations W_j which are initially applied to the unit square S_0:

$$S_0 = \{(x_1, x_2) \in \Re^2 \mid (x_1 \geq 0), (x_1 \leq 1), (x_2 \geq 0), (x_2 \leq 1)\}$$

$$W_1 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, 0, 0\right] \quad W_2 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, 0, \frac{1}{3}\right] \quad W_3 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, 0, \frac{2}{3}\right] \quad W_4 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right]$$

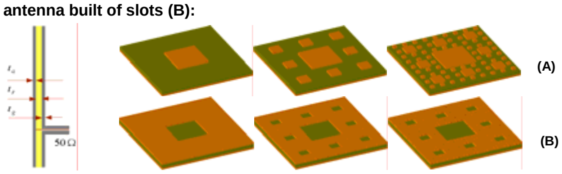
$$W_5 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right] \quad W_6 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right] \quad W_7 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{2}{3}, 0\right] \quad W_8 = \left[\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{1}{3}, 0\right]$$

The first four SC iterations (i = 1, …, 4):



2. Modeling the Antenna

The project considers two types of antennas: an antenna built of patches (A), and an antenna built of slots (B):



The algorithms which are based on the affine transformations and are capable of generating the geometries of both types of antennas have been implemented as completely parameterized models for *QuickWave-3D*, the 3D conformal FDTD electromagnetic simulator. The models were used to calculate the return-loss characteristics of both antennas.

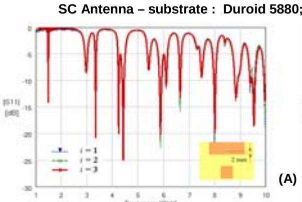
Validating the *QuickWave-3D* Model

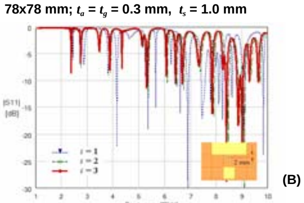
Resonant Frequencies of the Patch SC Antenna (i = 3) – substrate: Duroid 5880 (ε = 2.2 – i0); 196x196 mm; probe: 22.9 mm from the edge of the main patch; $t_a = t_g$ = 1 mm, t_s = 1.6 mm

Measured [1]	1.5				4.5		
Calculated [2]	1.49	3.01	3.36	4.28	4.48	5.49	5.96
Our model	1.48	2.98	3.34	4.25	4.43	5.43	5.89

SC Antenna: Patches & Slots

SC Antenna – substrate : Duroid 5880; 78x78 mm; $t_a = t_g$ = 0.3 mm, t_s = 1.0 mm





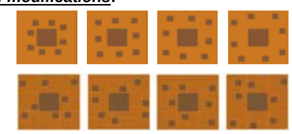
NB: The patch antenna is unlikely to be controlled by the fractal iterations, while the *slot antenna does allow for this type of control*.

3. Fractal Modifications

This project focused on trying to improve the multiband and wideband properties of the SC antenna. The original results from this work concern the process of modifying the SC fractal geometry for i = 2.

Specifically, the project considers *two types of modifications*:

1) **the systematic outward movement and rotation of the smallest elements with respect to the center element.**
2) **the pseudo-random motion of the smallest elements within their immediate sub-domain.**



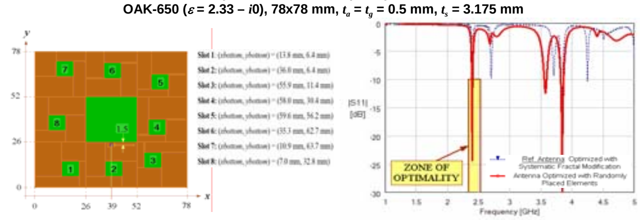
4. Optimization Problem & Results

To perform the numerical FDTD-backed optimization, we use a *neural network procedure* [3] and solve the following optimization problem:

Find the geometry of the return loss less than an assigned level S_* in a specified frequency range (f_1, f_2).

NB: For the modification which considered the pseudo-random motion of the smallest elements, the optimization procedure produced a configuration which had a return-loss characteristic which was *well within the zone of optimality*:

Modified SC Slot Antenna Optimized for the Frequency Band 2.4 to 2.485 GHz substrate: OAK-650 (ε = 2.33 – i0), 78x78 mm, $t_a = t_g$ = 0.5 mm, t_s = 3.175 mm

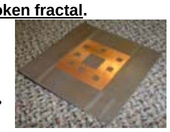


5. Conclusions

- **The developed 3D model of the SC antenna was proven to be adequate.**
- **Variations in the geometry of the SC fractal antenna allow for very limited control over the frequency characteristics; the SC pattern is *not optimal* in terms of bandwidth/wideband performance.**
- **It was possible to "tune" the antenna at particular frequencies and increase the widths of the frequency bands by using the pattern of a broken fractal.**

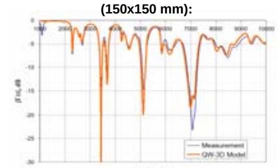
The results constitute a complete original engineering design of the broken SC fractal antenna.

The sample of the optimized antenna has been produced and experimentally tested by the project's sponsor:



- **The measured data and calculated results produce *almost identical* return loss characteristics**

Measured and Calculated Return-Loss Characteristic of the Manufactured Antenna (150x150 mm):



Given specific optimality constraints, the developed method can be used to determine the optimal geometry which has a resonant frequency below a certain return loss level and within a certain frequency range.

References

[1] R.V. Hara Prasad, Y. Purushottam, et al., *Electronics Letters*, vol. 36, No 14, pp. 1179-1180, 2000.
[2] Z. Du, K. Gong, et al., *Electronics Letters*, vol. 37, No 13, pp. 805-806, 2001.
[3] E.K. Murphy and V.V. Yakovlev, *Proc. 9th AMPERE Conf. on Microwave and High Frequency Heating, Loughborough, U.K, September 2003*, pp. 197-200.

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