

## **An Outcomes Oriented Approach to Calculus Instruction**

John Goulet, Department of Mathematical Sciences, WPI  
(appears in: Journal of Engineering Education, April 2001 (Vol. 90, No.2))

### **I. Background**

During the Fall of 1999, I taught Calculus IV, Multivariable Calculus, in an outcome or "goals" oriented manner. This note explores the reasons, implementation and results of that effort. The course had 55 students, all engineering or science majors.

### **II. Perceived Relation to ABET Criteria**

While I am not in a department which directly falls under the ABET umbrella, I can still foresee the point where more specific demands of departments like Mathematics will be made by ABET. At this point, students must take, according to ABET, calculus and other mathematics courses, and be able to apply concepts to other subjects, presumably engineering. In short, mathematics courses exist in a supporting role. Students are not simply trying to accumulate a sufficiently long list of mathematics courses that they have passed. They are trying to take mathematics courses which provide specific concepts and capabilities to help them perform better in engineering courses.

The "goals approach" provides a logical and straightforward means of going about this. It begins with communication between myself and instructors of science and engineering who will inherit the students. The mathematical needs of the students relative to those subsequent courses, whatever they are, are identified. They become part of the desired outcomes of my mathematics course. Doing what it takes to see they are achieved becomes the job of myself and my students. This approach really has little to do with calculus per se. It could easily be used with any freshman or sophomore level course, whether that is in mathematics, physics, chemistry or

engineering. The fundamental consideration is: if an instructor in a higher level course having my course as a prerequisite has students from my course, then they ought to have a say in the preparation of those students, and to know what to expect when one of them arrives in their course in terms of capabilities, not just that they have simply passed the course. They would not care *how* those capabilities were obtained, only *that* they were. That seems consistent with a goals oriented approach. The issue of EC 2000 and assessment at WPI has been extensively addressed in reference 1, with only the component of the course not explored. Since the course is the fundamental building block of a students experience, this approach may have significance in assessing ABET criteria.

### **III. Implementation**

#### A. Establish Goals

First of all, the departmental course requirements were reviewed and an initial list of goals generated. The departmental requirements were more along the lines of material to be studied than outcomes. (This problem seems to be frequently true when people come to list goals for the first time - a transition has to occur from phrases like "will study" to "will be able to do"). The initial list of goals was then reviewed by the Associate Department Head of Mathematical Sciences. Next, this particular course had a clear supporting relation to a Physics course in Electricity and Magnetism, being taken concurrently by all students enrolled (what WPI refers to as a "bridge" course). Its outcomes were then reviewed and refined relative to the needs of the Physics course, specifically goals 1-6 which relate directly to electric fields and voltages, in conjunction with the instructor of the physics course. Once the final list was in place, they were handed out to all students during the first class, and perhaps more important, considered as a fixed target by both students and instructor.

The following were stated in the course syllabus, reference 2 :

Table 1

Goal #	Statement
1	set up and interpret line integrals of vector functions
2	compute and interpret partial derivatives
3	determine potential functions when possible
4	evaluate line integrals using potential functions
5	construct (describe) level curves of functions of two variables $f(x,y)$
6	apply level curves of $f(x,y)$
7	set up double and triple integrals in Cartesian coordinates
8	set up triple integrals in Cylindrical coordinates
9	set up triple integrals in Spherical coordinates
10	determine and classify critical points of $f(x,y)$
11	understand and apply Least Squares curve fitting
12	differentiate using the chain rule
13	compute and apply gradients of scalar functions
14	compute and apply directional derivatives of scalar functions
15	use and interpret directional derivatives

None of this is cast in stone. At another time, a different list of goals might result for the same course. But once in place, it then becomes the job of the students and instructor to see that they are met, by as many students as possible. The *method(s)* (lecture, cooperative,...) chosen are seen as a means to an end, and various approaches in an outcomes oriented environment have been reviewed in reference 3.

How do these goals relate to ABET criteria? Being a freshmen level course, as one might expect, they are strong in the knowledge/comprehension/application areas and do not really address analysis/synthesis/evaluation/valuation areas which more advanced courses such as Differential Equations or Modeling might. Writing is an open issue which needs to be addressed in the future. The general issue of outcomes and ABET criteria is discussed in reference 4.

#### B. Establish a Grading Scheme for the Course

Having done this, the criteria for passing the course could now be established: *one had to demonstrate minimal competence at each and every goal*. Do that, and the course is passed. Fail to do that for one or more goals and the course has not yet been passed.

1) How does one demonstrate "minimal competency" at a goal?

Weekly exams were given, with questions specific to the goals. If one got 70% of the possible points on a goal oriented question, then that goal was considered passed, and "checked off" . This places an important stipulation on the instructor, or whoever is doing the grading. When grading an individual problem which measures achievement of a goal, beyond simply assigning points, one must make the fundamental determination: *does this work represent a minimally acceptable level for this goal?* If so, then give at least 70% of the points. In mathematics, this may cause one to rethink how grading is done.

2) What if one did not pass a goal?

Here, they had to find out *why* they failed to pass the goal, and a retest of the goal was offered on a weekly basis. (This offered valuable feedback to the instructor in the process of reviewing their substandard work with them).

When they finally passed the goal, the grade obtained on that passing effort was substituted for the original, failing grade. A hypothetical example follows: a student gets 12 out of 30 on the question pertaining to goal #3 (less than 21 points or 70% of 30 so not passing). The overall quiz score might very well be quite good, but irrelevant to a goal was not being met. The student meets with the instructor to review "what went wrong", with suggested things to study and do as a result. They then take a makeup quiz on just that single goal. Suppose on the makeup question they now get 80% of the possible points. The goal is passed and the original score of 12 points on the question has 24 replace it (80% of 30), raising their quiz score by 12 points. Such a

procedure is viewed as necessary and consistent with the "outcomes" philosophy. To the student, it simply might mean "It's where you have gotten to, not how you got there".

All of the above serves to determine who passes the course. WPI requires a letter grade of A,B or C, so at this point, for students who have passed, the letter grade is assigned based upon *how well* they passed the goals. Quizzes, and the final exam are combined to produce a score for the entire course. An A is for 90 or above, B for 80-89, and C for everything else.

#### **IV. Incremental Improvement**

In a traditional course setting, once a topic has been studied and an exam given, while that material may well be referred to in the future, it is essentially history . If a student did not do well on that portion, their grade reflects it, but in any event, we all move on.

In the goals environment, we look at specific goals as involving several steps of a feedback loop whose purpose is eventual achievement of the goal. First, one works on homework. Assuming one does not get all problems right, an attempt is made to identify where things bogged down, rather than simply saying "I couldn't get number 8". Once that is fixed, then the person might still fail the goal on a quiz. We then break down that work and see what went wrong. Usually, on a second quiz, the person has it all together and the goal is met. The basic approach of problem identification at each step, and solution lead to incremental improvement, and eventual satisfaction of the goal. Perhaps more important, this kind of thinking is no different than one employs in many areas of engineering.

An example is goal, #9, setting up a triple integral in spherical coordinates. This is, by statistical evidence, the most difficult goal in the course. By the time students have satisfied it, they have seen that it is not really a single problem, but a collection of smaller problems, successfully

solved by careful analysis of right triangles, detailed sketches, and correct use of the concept of coordinate.

## **V. Results**

### A. How did the students respond to it and feel about it?

I was initially skeptical about the reaction I would get from students. Almost all of them come from traditional educational settings, and were successful achievers in those settings. Here I was making life potentially more difficult. As an example, in the hypothetical case above, a student could easily get a B on a quiz ( based on a score between 80 and 90) yet have failed a goal and thus have to go back and do more work to address the failed goal.

The results were just the opposite. Based on written comments on course evaluations at the end of the course, as well as informal discussions, they were quite enthusiastic about

- having clear cut objectives stated and pursued
- the chance to get better at something if not initially successful
- the means to improve their grade

Some data supports the first claim. WPI course evaluations have as their initial consideration "The instructor established clear objectives for the course". Students respond with either Strongly Disagree (SD), Disagree (D), Agree (A) or Strongly Agree (SA).

I had taught the same course the term just prior to implementing a goals oriented approach. My own data from that question had 36% A and 64% SA from a class of 49 students. I had used traditional grading methods at that time. From the evaluations from the 55 students in the goals oriented version, I got 6% A and 94% SA.

The WPI course evaluation form has opportunities for written responses . Not all students use this portion. Since the Fall of 1999, I have used the goals approach with over 150 students in

three courses. Many have taken the opportunity to mention the goals approach. *All* were positive. Samples below are to the question "What did you particularly like about this course/lab?":

"The fact that this course was goal oriented was VERY helpful. All courses should be taught this way!"

"The goals are a great idea. We know exactly what to learn, and exactly which aspects to study."

"Everything was clearly stated. I knew exactly what I had to do."

"The goals of the course were very clear and we needed to understand all of them to pass."

"This course was well structured and it was understood (as to) exactly what was expected".

"The instructor is far more concerned with education than grades".

"The goal system".

"The professor has a very good teaching method - the goals system".

"How the professor spent time going over the material again and again until fully understood".

"The goals based course structure".

"The goal orientation was much more beneficial than the grade orientation".

"the goals concept. It made my life harder but I feel like I learned more."

"the goal based learning method was very efficient and is a great way to evaluate a student's understanding."

## B. How did the Instructor Feel?

### 1) Positive Aspects:

- overall clarity of purpose and goal were substantially better
- the course could be put in a more meaningful position relative to other courses and curricula

- the chance to *improve* upon students work, and to understand *why* it had shortcomings was most valuable, as part of the process. The opportunity for the *instructor* to improve is substantially greater and a valuable part of this process.
- the course could be put in a more meaningful position relative to other courses and curricula

## 2) Negative Aspects:

- need for better information management and flow. Much more data is involved (I have the point total for every *question*, not just every exam), and students need an easy mechanism to frequently see "where they stand". There is, however, web based courseware which will be used in the future and should alleviate much of this.

During the initial trials of this approach, at two different points, I handed out slips of paper which had the following brief information: the students name and what goal(s) they had not met to date. The web software referred to would make this information continually available.

## C. Did Improvement Occur?

I made the point earlier that we kept working on each goal until it was met. In a traditional course, the topic would have been abandoned after the initial attempt, so I am taking the view that whenever someone passed a goal on a second or third attempt, then improvement has occurred relative to what would have happened in a traditional setting.

In the most recent occurrence of this assessment method, there were 15 goals and 35 students, so 525 total requirements of a goal by the class. In 69 instances, a goal was not met on the first try, but was eventually met, indicating improvement to a satisfactory level.

## **VI. Conclusions**

Establishment and pursuit of course outcomes is appealing and beneficial to all involved parties: the institution, the department, the instructor and the students. I would agree with the King's College experience, described in reference 5 that a fundamental move is possible to learning, for all involved parties. Clarity, focus and communication are primary reasons for this. The approach taken here is, in many ways, a gentle conversion to an outcomes based system, for it modifies a traditional approach rather than scrapping everything and starting over with many potential problems. This might allow it to be transplanted more easily to other existing courses. Finally, it allows a course to exist in a clear cut and unambiguous way as a component of a curriculum, rather than as an isolated activity.

## References

1. Schacterle, L., "Outcomes Assessment at WPI: A Pilot Accreditation Visit Under Engineering Criteria 2000," *Journal of Engineering Education*, vol. 87, no.2, 1998, pp. 115-120.
2. <http://www.wpi.edu/~goulet/calc4P/syll.htm>
3. Demetry, C., "Outcomes Assessment of Educational Approaches: Examples for Materials Science Educators," *Journal of Materials Education*, vol. 20 (1&2),1998, 1-8.
4. Besterfield-Sacre, M., J. McGourty, R. Miller, B. Olds, G. Rogers, L.J. Shuman, and H. Wolfe, "Defining the Outcomes: A Framework for EC 2000" *IEEE Transactions on Engineering Education*, vol. 43, no. 2, May 2000.
5. Farmer, D., "Course-Embedded Assessment: A Catalyst for Realizing the Paradigm Shift from Teaching to Learning," *Journal of Staff, Program and Organization Development*, vol. 16, no. 4, 1999, pp. 199-120.
6. Rogers, G.M., "EC2000 and Measurement: How Much Precision is Enough?," *Journal of Engineering Education*, vol. 89, no.2, 2000 pp. 161-175.

## Appendix

What follows are sample test questions used to assess satisfaction of goals 1 and/or 2:

### Problem 1

The curve  $C_1$  is parameterized by the vector function

$$\mathbf{r}(t) = t^2 \mathbf{i} + 2t^2 \mathbf{j} + 3t^2 \mathbf{k} \quad \text{with } 1 \leq t \leq 4$$

- for a force field  $\mathbf{F}(x,y,z) = (x+z) \mathbf{i} + z \mathbf{j} + xy \mathbf{k}$
- set up an integral to give the **work** to move along the curve  $C_1$  (30 points)
- consider a second curve,  $C_2$ , described parametrically by

$$\mathbf{r}(t) = (1+5t) \mathbf{i} + (2+10t^2/3) \mathbf{j} + (3+5t^3/3) \mathbf{k} \quad \text{for } 0 \leq t \leq 3.$$

For the same force field as in part **b**, it has been computed that the line integral of  $\mathbf{F}$  along this curve comes out to 10.5.

$$\int_{C_2} \vec{F} \bullet d\vec{r} = 48.26$$

- Based upon this, what can you say about the field  $\mathbf{F}$  as far as being *conservative* or not?

Please **give all steps** used in your reasoning for full credit. An unsupported answer will not receive any credit. (20 points)

### Problem 2

Suppose the function  $f(x,y) = 10\sin(x^2 + 2xy + 3y^2 + \pi/4) + 70$  is the temperature, in Fahrenheit, at a point  $(x,y)$  on the national weather map, where  $x$  and  $y$  are in units of 100 miles, respectively East and North of Chicago.

a. what is  $f_x$ ?  $f_y$ ? (25 points)

b. Suppose at a particular point  $(x_0, y_0)$  it has been calculated that  $f_y(x_0, y_0) = 2.5$ . How would you interpret this information? Be as specific as possible in your answer. (25 points)

**Note:** Problem 1 may satisfy course goal #1. Problem 2 may satisfy goal #2.