Performance of a Precision Indoor Positioning System Using a Multi-Carrier Approach

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BIOGRAPHY

Dr. David Cyganski is a Professor in the ECE Department at WPI where he performs research and teaches in the areas of linear and non-linear multidimensional signal processing, communications and computer networks, and supervises the WPI Convergent Technology Center. He is an active researcher in the areas of radar imaging, automatic target recognition, machine vision and protocols for computer networks. He is a coauthor of the book Information Technology: Inside and Outside. Prior to joining the faculty at WPI he was an MTS at Bell Laboratories and has since held the administrative positions of Vice President of Information Systems and Vice Provost at WPI.

Dr. John A. Orr is a Professor in the ECE Department at WPI where he was department head from 1988 to 2003. Dr. Orr performed some of the earliest work on fading effects in CDMA communications systems. Dr. Orr was a member of technical staff at Bell Laboratories involved in the systems engineering of the video telephone network. More recent research interests include digital signal processing, automatic target recognition, and real-time data acquisition and processing. He is a coauthor of the book Information Technology: Inside and Outside. Dr. Orr is currently on sabbatical at Stanford University pursuing work in the area of GPS systems.

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ABSTRACT

System performance aspects of the Multi-carrier Ultrawideband (MC-UWB) approach to positioning are presented. This system implements a wideband or ultrawideband ranging signal in the frequency domain, retaining the frequency coordination and spectral allocation benefits of conventional systems while including signal characteristics which permit resolution of multipath signals in indoor or other difficult RF propagation environments. The overall system architecture was introduced in a previous paper. This paper presents the analysis of system performance as a function of the major parameters, including bandwidth, frequency component spectral spacing, transmitted signal power, and range. These results facilitate the design of MC-UWB positioning systems for specific environments.

INTRODUCTION

In a previous paper [1] the authors described a novel means for precision personnel location based upon a multi-carrier signal constructed in a manner similar to orthogonal frequency division multiplex. The individual carriers span a wide bandwidth (with very low occupancy of the total band), leading to the designation Multi-Carrier Ultrawideband (MC-UWB) for this technique. The MC-UWB signal structure and modern spectral analysis solution techniques offer significant advantages for the problem domain of precision personnel location. This signal structure offers high precision location at a cost of essentially infinitesimal bandwidth thanks to its sparse line spectral content. The ability of modern spectral analysis to determine the frequencies of arbitrarily-spaced components of a signal permit a super-resolution solution for time difference of arrival information from a signal despite large-amplitude multipath components.

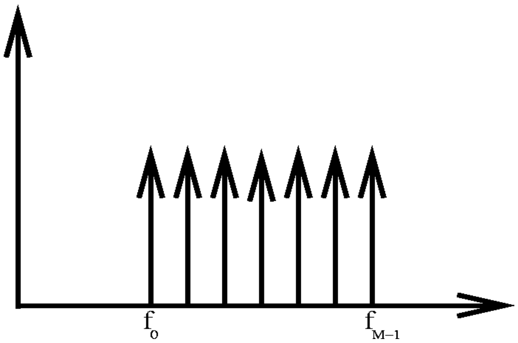

Figure 1: The simplest MC-UWB signal consists of N carriers with equal spacing and equal magnitudes.

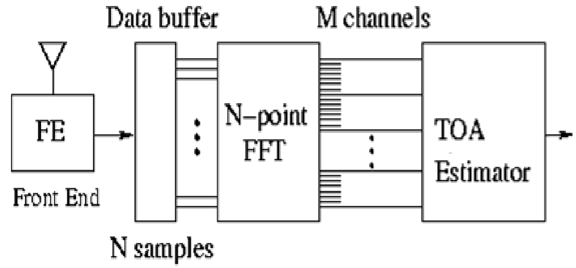

Figure 2: An example of a direct conversion implementation of a reference node receiver based upon FFT processing of the sampled UWB signal.

The prototype application for this system is precision, real-time personnel location for first responders inside buildings (such as law enforcement officers and firefighters, and extending to military, corrections, and similar personnel). This represents an extremely challenging and hostile environment for positioning systems, but such systems would meet an urgent need. The problem to be solved is distinguished from the GPS situation in several ways, some of which simplify the problem (such as the relatively small range of operation and the restricted number of simultaneous users) and some of which complicate the situation (such as the indoor, multipath environment, the need for precision on the order of 10 cm, the need for rugged, light weight, long-lived rover units, and the ad hoc nature of each response situation).

Positioning precision and multipath immunity are usually associated with impulse-type ultra wide bandwidth signals whose spectral footprint presents spectral allocation and/or inter-service interference problems. The signal structure and TDOA (time difference of arrival) recovery approach of this paper avoid these problems with a ranging signal that is amenable to spectral assignment. This approach also takes a form that separates the notions of spatial precision and multipath immunity from those of bandwidth and temporal confinement of the pulse.

The simple form that the mobile node takes, that of a transmitter of a single, periodic signal with no time synchronization requirements, immensely lowers the cost of equipping personnel and materiel as compared with systems that require complex receivers or transceivers. The entire system is amenable to simple software radio implementation as demonstrated by a prototype implemented acoustically. Since the signal structure and system implementations with an audio channel are one-to-one with those used for RF OFDM communications systems, this opens the opportunity to integrate precision location into existing OFDM systems and/or to provide OFDM communications channels in any such realization of a precision locator.

In this paper a method for estimating system performance in terms of transmitted power, range of operation, and and other operational and signal structure parameters is derived. The estimate is obtained by combining the Cramer Rao Bound for the Time of Arrival (TOA) estimate for the multi-carrier signal and a simple position error estimator previously obtained for location estimation from TDOA information. The intent of this paper is to find the general form of the result and approximate performance parameters in order to understand the relative importance of design parameters such as sensor spacing, transmitted power, fractional bandwidth, maximum signal frequency, number of carriers, etc. Hence, only the best case scenario for the signal (no multipath components or channel distortion) and precise synchronization of all component clocks are assumed here. Future work will remove these limitations.

MULTI-CARRIER SIGNAL STRUCTURE

As described in the previous paper, the new method is based transmission of a continuous multi-carrier signal of the form

$$s_c(t) = \sum_{m=0}^{M-1} A_c e^{2\pi j(f_0 + m\Delta f)t + \phi_m} \qquad (1)$$

comprising M sinusoidal carriers with frequency spacing Δf and arbitrary phases ϕ_m. For simplicity we will consider only the case of baseband signal generation at the transmitter and a direct conversion (similarly baseband) receiver. Discrete Fourier transform processing may be used to implement this architecture as shown in Fig. 2. Let N be the number of time samples that are generated by the transmitter's IDFT and then repeatedly transmitted to form the continuous output waveform and f_s be the sampling rate.

Furthermore, we will consider only the case of real signals and hence describe our signal by

$$s(t) = s_c(t) + s_c^*(t) \qquad (2)$$

$$= \sum_{m=0}^{M-1} A\cos(2\pi(M_0 + mK)\delta ft + \phi_m) \quad (3)$$

in which we can express the parameters of the previous equation in terms of the sampled signal system as $\Delta f = K\delta f$ and $f_0 = M_0\delta f$, where $\delta f = \frac{f_s}{N}$ is the DFT frequency sample separation.

If this signal is received at reference sites, with distances d_k from the source giving rise to propagation delays $\tau_k = d_k/c$, then each carrier component is shifted by a phase shift which depends both upon τ_k and the carrier's frequency.

It was previously shown [1] that if we assume that the reference sites have synchronized clocks, while the source clock has some unknown offset t_0, this offset induces another unknown phase shift dependent on carrier index $\psi_m = 2\pi(f_0 + m\Delta f)t_0$.

Let's now assume that the sampled received signal at the k_{th} reference is of the form

$$r_k(n) = s_k(n) + n_k(n) \quad (4)$$

for $n = 0..N-1$ with DFT

$$R_k(m) = S_k(m) + N_k(m). \quad (5)$$

The phase difference between adjacent carriers for the signal received at reference node k, $S_k(M_0 + mK)$ for $m = 0..M-1$, corrected for the known phases ϕ_m satisfies

$$\Delta\theta_k = \Phi_{mk} - \Phi_{m-1,k} - \phi_m + \phi_{m-1} \quad (6)$$

$$= -2\pi\Delta f\tau_k + \psi_m - \psi_{m-1} \quad (7)$$

$$= -2\pi\Delta f\tau_k + m\Delta\psi \quad (8)$$

modulo 2π.

Finally, the difference of the phases obtained as above for carrier m at two sites, q and r, is $\theta_{qr} = \Delta\theta_r - \Delta\theta_q$, from which we can recover the Time Difference of Arrival (TDOA) of the signals at those sites,

$$\Delta\tau_{qr} = \frac{-\theta_{qr}}{2\pi\Delta f} = \frac{d_2 - d_1}{c} \quad (9)$$

in which c is the velocity of the wave in our medium.

Now, since our signal is periodic, with period $T = \frac{1}{\Delta f}$, the solution suffers a time (range) aliasing ambiguity. However, thanks to the limited spatial scope of the location problem we defined, by choosing Δf sufficiently small, we can make our TDOA solution unambiguous throughout a ranging cell which is defined by the locus of points within distance $R = \frac{c}{\Delta f}$ of any receiving site.

Thus the problem of TDOA estimation for this signal reduces to the estimation of the consistent phase change between adjacent carriers in the received signal, $\Delta\theta_k$. That is, the phasor amplitudes of the M carriers (after compensation for the known phases ϕ_m) have the form $c_1 e^{2\pi j\Omega m}$ and we seek the frequency sample phase progression coefficient $\Omega = \Delta\theta_k$. This problem is effectively a sinusoidal frequency estimation problem applied to the phasor valued signal spectrum.

Throughout the following discussion there is ample opportunity for confusion owing to the fact that:

1) our sampled multi-carrier time domain signal consists of a sum of sinusoids;
2) the signal's DFT sampled frequency domain representation presents a line spectrum whose phasor amplitudes at the subset of carrier frequency related indices, after normalization by the initial transmitted phase values, form a progression equal to that of samples of a single complex sinusoid of "frequency" determined by the path delay.

Thus the appearance of sinusoids and related frequency concepts on both sides of the DFT can easily lead to confusion regarding whether or not a given statement applies to the time or frequency domain samples. In the following we will refer to samples of the DFT frequency domain as frequency sample values. When referring to just those frequency samples that correspond to carrier frequencies, we will refer to the carrier subset frequency samples.

CRAMER RAO BOUND FOR SINUSOIDAL FREQUENCY ESTIMATION

The problem of estimating the frequencies of multiple, arbitrarily spaced, sinusoidal signals from a noisy linear combination has a long history with the first general method having been proposed by Prony in 1795 [2]. Modern analytic and computational methods are now known collectively as Modern Spectral Analysis [3].

Rao and Arun [4] provide an expression for the Cramer Rao Bound (CRB) for estimation of the frequency coefficient $\Delta\theta_k$, which expressed in terms of variables defined above is

$$E\left\{\Delta\theta_k^2\right\} = \frac{6}{M^3}\frac{\sigma_n^2}{|c_1|^2}, \quad (10)$$

in which σ_n is the standard deviation of the additive white Gaussian noise (AWGN) associated with each frequency sample and c_1 is the amplitude of the complex sinusoid formed by the frequency sample values. We must now obtain the latter values in terms of the parameters of the original multicarrier signal.

Since our received signal is assumed to consist of M real sinusoids with amplitudes A, the total received signal power is given by,

$$P_s = E\left\{s_k(n)^2\right\} = \frac{1}{2} MA^2. \quad (11)$$

By Parseval's theorem, the power in each of the $2M$ frequency samples in the receiver DFT output corresponding to the M carriers must be

$$P_S = E\left\{S_k(n)^2\right\} = N\frac{P_s}{2M}. \quad (12)$$

To simplify the analysis we will assume that P_s and P_S are invariant with respect to the reference index. In general this would not be true given the different attenuations that would be suffered by the signal traveling the different distances to the various receivers. However, analysis that would include the equalization of the signal amplitudes and hence introduce full treatment of the spatially colored noise in later steps would obscure the flow of this exposition.

Since the frequency samples in the case of a single received multi-carrier input form a complex sinusoid, and since the power associated with these samples is given by P_S, above, the amplitude $|c_1|$ in the CRB expression is given by

$$|c_1| = \sqrt{\frac{NP_s}{2M}}. \quad (13)$$

Likewise, we can obtain the variance of the noise component of the carrier index frequency space samples. Given AWGN noise with single-sided power spectral density of N_0 watts/Hz, then the total noise power in the Nyquist band and hence the variance of the noise component of any time sample is

$$P_n = \frac{N_0 f_s}{2}. \quad (14)$$

Since the noise is assumed to be white, by Parseval's theorem we have that the variance of the noise component of each frequency space sample is identical to the variance of the time-domain noise sample variance, hence, $P_N = P_n$. Hence the noisy complex sinusoid formed by the carrier subset of frequency samples takes the from of a sinusoid with power P_S imbedded in noise with variance P_N, where each of these values can be represented in terms of the underlying time-domain power levels given by P_s and P_n given above.

We can now insert the above expressions into Rao and Arun's CRB to obtain

$$E\left\{\Delta\theta_k^2\right\} = 12\frac{P_n}{M^2 N P_s}. \quad (15)$$

Since the time delay associated with the phase progression parameter is given by

$$\tau = \frac{\Delta\theta}{2\pi K\delta f} = \frac{N\Delta\theta}{2\pi K f_s} \quad (16)$$

we can write the CRB bound for the variance of the time estimates as

$$\sigma_\tau^2 = 3\frac{NP_n}{\pi^2 K^2 f_s^2 M^2 P_s}. \quad (17)$$

While, for simplicity, the above discussion was carrier out in the context of a direct conversion receiver wherein the input signal is simply sampled, we can easily place these results into a form that represents the performance bound for delay optimization in the more general case of a heterodyne receiver in which a sub-band of width B Hz is sampled at rate $f_s = 2B$ for a time window of $T = \frac{N}{2B}$, yielding

$$\sigma_\tau^2 = \frac{3}{8}\frac{N_0}{\pi^2 B^2 T P_s}. \quad (18)$$

It is notable that in this form all apparent dependence upon the number of carriers disappears with the only significance of this parameter being the size of the spatial aliasing cell.

CRB FOR POSITION ESTIMATION

Position estimation for the system is carried out by solving the Time Difference of Arrival (TDOA) problem. A CRB for this estimate for the case of Gaussian noise perturbation of the TDOA values with a given TDOA covariance matrix is given in [5]. While this expression was used to perform validation of our position estimator subsystem, its complexity diminishes its value as a means of gaining insight into the end-to-end performance of this location system. For that reason, for the remainder of this paper we will abandon the location estimate CRB in favor of a simple performance estimate (not a bound) given by Bard [6].

Bard showed that if A is a geometry matrix given the positions of S-1 of the S reference receivers relative to receiver 0, that is,

$$A = \begin{bmatrix} x_1^T - x_0^T \\ x_2^T - x_0^T \\ \\ x_{S-1}^T - x_0^T \end{bmatrix} \quad (19)$$

then, if the target is far from all receivers, the position estimation variance is given approximately by

$$\hat{\sigma}_r = c r_0 \sigma_{\delta\tau} \sqrt{\left(Tr\left\{(A^T A)^{-1}\right\}\right)}, \quad (20)$$

where r_0 is the overall distance of the target from the group of reference sensors and $\sigma_{\delta\tau}$ is the TDOA error standard deviation for errors that are modeled as independent AWGN. While Bard derived this for the case of large sensor to target separation, we have found that this expression yields order of magnitude correct results even when the target is amidst the sensors, providing a useful summary performance estimate.

Examination of this expression reveals that location performance degrades as a function of the size of the TDOA estimate errors and the distance of the target from the sensors and also that performance is a function of sensor geometry.

RECEIVER SIDE COMBINED LOCATION PERFORMANCE

We can now combine the results of the last two sections to obtain the performance of the locator with respect to received signal characteristics. Furthermore, we will substitute a particular sensor geometry into the equation in order to obtain a simple parameterized model for purposes of building our intuition about performance. For this purpose we will use the geometry matrix given by

$$A = \begin{bmatrix} 0 & 0 & h \\ w & 0 & 0 \\ w & 0 & h \\ 0 & w & 0 \\ 0 & w & h \end{bmatrix} \quad (21)$$

wherein six sensors are used, three placed in the z=0 (ground level) plane in a triangle comprising the origin and two points w meters offset in the x and y directions from origin. The remaining three sensors are each h meters above the first set. Likewise we fix the position of the target at approximately w units from all the sensors, that is, essentially at the far corner of the cube, three sides of which are defined by the sensors.

Inserting this geometry and the previously obtained CRB for the time of arrival estimates we obtain

$$\hat{\sigma}_r = \frac{1}{8} \frac{c\sqrt{6N_0}\sqrt{5h^2 + 2w^2}}{\pi\sqrt{P_s T Bh}} \quad (22)$$

wherein we have ignored the fact that the TDOA errors are actually differences of TOA errors, and hence not independent as we seek only an order of magnitude estimate of performance.

RF CHANNEL PERFORMANCE

Now we consider the performance of the RF channel over which these signals transit. As is well known, the Friis transmission formula expresses the received power, P_s, in terms of the transmitted power P_{trans}, the antenna gains, etc.:

$$P_s = \frac{P_{trans} G_{trans} G_{rec} \lambda^2}{16\pi^2 r_0^2}, \quad (23)$$

in which r_0 is, as before, the distance from the target to the various sensors, and G_{trans}, G_{rec} are the transmitting and receiving antenna power gains, respectively. If we take the antennas to be omnidirectional, then the antenna gains both become unity and may be omitted from the following equations. Finally, λ is the wavelength of the RF signal in meters.

Given a received signal with a noise figure, NF and antenna temperature of T_a degrees Kelvin, then we can express the noise power spectral density of the received signal as

$$N_0 = 4 k_B T_a 10^{NF/10},$$

where k_B is Boltzmann's constant. We can now substitute these expressions for the noise power density and the received signal power into our earlier equation for the estimated standard deviation of location error. We will assume the particular geometry of target and sensor chosen before, omnidirectional antennas, a front end noise figure of 3dB, antenna temperature of 290 K and a wavelength equal to the shortest wavelength in the signal (hence worst case attenuation, that associated with f_{max}). With these substitutions we obtain

$$\hat{\sigma}_r = 2.19 \cdot 10^{-8} \frac{\sqrt{5h^2 + 2w^2} f_{max} w}{Bh\sqrt{P_{trans} T}}.$$

This can be further simplified by noting that $E = P_{trans} T$ is the energy in each period of the transmitted waveform and that $F = \frac{B}{f_{max}}$ is the fractional bandwidth of the signal. Thus, for the case under study:

$$\hat{\sigma}_r = 2.19 \cdot 10^{-8} \frac{\sqrt{5h^2 + 2w^2}\, w}{Fh\sqrt{E}}.$$

What is remarkable about this result is that performance is not directly a function of the number of carriers used or the band in which these carriers are deployed, of the speed of the signal in its medium (c), but rather only of the geometry, the fractional bandwidth of the signal and the energy per signal period. Also remarkable is the high precision that can be achieved with relatively modest power levels and fractional bandwidths if the sensors and target are closely co-located. This is best seen by

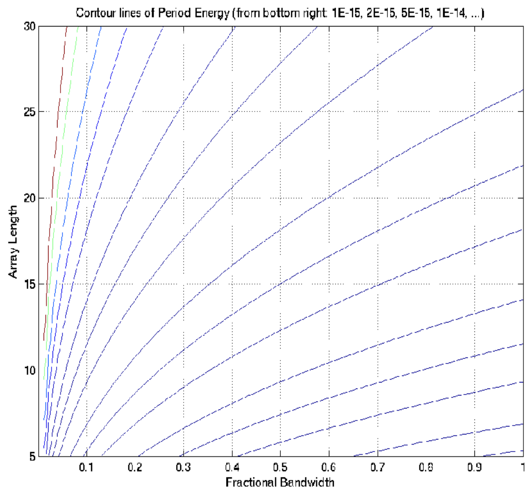

Figure 3: Nomograph generated from our design equations for the case of the 6 sensor geometry given in the text and a targeted location estimate standard deviation of 10 cm.

generating a system design nomograph from these equations which targets a given performance level.

In Fig. 3 we have a nomograph that relates the required values of w, E and F for the case of a targeted location standard deviation of 10 cm and a height separation of sensors of 5 meters.

We have confirmed the above estimates via both Monte Carlo tests of an end-to-end simulation of this system and by comparison with measured results taken from an audio signal based demonstrator (which was described in [1]). For example, in a test conducted with N=8192 time samples per signal period, M=101 carriers and an SNR=-11.2 dB, the predicted standard deviation of location estimates about the mean was 0.0739 inches while the experimental result was 0.0527 inches for 1000 trials.

CONCLUSIONS

Analysis of the system performance as a function of transmitted signal power and bandwidth and system geometry has yielded encouraging results and has produced performance equations which can be used for initial design of prototype positioning systems to achieve desired performance levels. Further work is underway to tighten the performance bounds and to include additional real-world aspects (such as clock synchronization errors) not included here.

ACKNOWLEDGMENTS

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