Performance Limitations of a Precision Indoor Positioning System Using a Multi-Carrier Approach

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BIOGRAPHIES

Dr. David Cyganski is a Professor in the ECE Department at WPI where he performs research and teaches in the areas of linear and non-linear multidimensional signal processing, communications and computer networks, and supervises the WPI Convergent Technology Center. He is an active researcher in the areas of radar imaging, automatic target recognition, machine vision and protocols for computer networks. He is a coauthor of the book *Information Technology: Inside and Outside*. He has held the administrative positions of Vice President of Information Systems and Vice Provost at WPI.

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Mr. Ryan Angilly recently completed his BS degree in Electrical and Computer Engineering at WPI. Mr. Angilly is a research assistant in WPI's Convergent Technology Center sponsored by the NIJ Precision Personnel Locator Project and is also concurrently the student program manager for the WPI PANSAT project, an AFRL/NASA-sponsored project that funds universities to design, build, test, and fly nanosatellites. He is currently working towards his Masters of Science degree in Electrical and Computer Engineering at WPI.

Mr. Benjamin Woodacre recently completed his MS degree in Electrical Engineering at WPI, where he served as a research assistant in the WPI Convergent Technology Center sponsored by the NIJ precision personnel locator project. His thesis examined variation of TDOA location algorithm performance as a function of geometry and techniques for system auto-calibration. Mr. Woodacre is currently a Ph.D. candidate at WPI.

ABSTRACT

The spatial resolution limits of the Multi-carrier Ultrawideband (MC-UWB) approach to positioning are presented. This paper directly and quantitatively addresses the underlying causes of performance degradation and system failure in the presence of multipath signals. While the results have been obtained in the context of the MC-UWB system, the predicted performance should be applicable to other signal structures with equivalent bandwidth and SNR. This system implements a wideband or ultrawideband ranging signal in the frequency domain, retaining the frequency coordination and spectral allocation benefits of conventional systems while including signal characteristics which permit resolution of multipath signals in indoor or other difficult RF propagation environments. The overall system architecture and other aspects of system performance analysis were presented in previous papers.

INTRODUCTION

In previous papers [1-3] the authors described a new means for precision position determination based upon a multi-carrier signal constructed with individual carriers spanning a wide or ultrawide bandwidth (with very low occupancy of the total band), leading to the designation Multi-Carrier Ultrawideband (MC-UWB) for this technique. The comb-like signal structure is analogous to the orthogonal frequency division multiplex (OFDM) technique employed for broadband communications and offers high precision location at a cost of essentially infinitesimal bandwidth thanks to its sparse line spectral content. The ability of modern spectral analysis to determine the frequencies of arbitrarily-spaced components of a signal permit a super-resolution solution for time difference of arrival information from a signal despite large-amplitude multipath components.

This system is being designed for the specific domain of precision personnel location in the indoor environment, in particular for fire, police, and other first responders. From the points of view of the multipath environment as well as the required precision, size, and cost, this represents an extremely challenging design task. However, it is clear that the availability of such a system would meet a long-standing and critical need. The problem to be solved is distinct from the GPS situation in several ways, but the solution will be complementary to the global extent and absolute positioning capabilities of the GPS system.

Impulse-type ultra wide bandwidth signals are typically proposed as a solution to this problem domain. However, the spectral footprint of impulse UWB signals presents spectral allocation and/or inter-service interference problems. The signal structure and TDOA (time difference of arrival) recovery approach of this paper avoid these problems with a ranging signal that is amenable to spectral assignment. This approach separates the notions of spatial precision and multipath immunity from that of temporal confinement of the pulse.

From the hardware point of view, this system has the advantage of a very simple mobile unit to be worn by the personnel to be tracked. The mobile unit is simply a transmitter of a single, periodic signal with no time synchronization requirements, immensely lowering the cost of equipping personnel as compared with systems that require complex receivers or transceivers. The entire system is amenable to simple software radio implementation and to integration with OFDM communications channels in any such realization of a precision locator.

This paper presents analytical, simulation, and field-test results which quantify positioning system performance in the presence of multipath and noise, parameterized by SNR and path length difference. The most significant result presented here is the relation between signal bandwidth and the minimal resolvable difference in path lengths between two signal paths (such as a direct path and an indirect path). As the requirement on minimal resolvable path length difference increases, required bandwidth also increases.

SIGNAL STRUCTURE AND DELAY DETERMINATION ALGORITHM

As described in previous papers [1-3], the new method is based transmission of a continuous multicarrier signal of the form

$$s_c(t) = \sum_{m=0}^{M-1} A_c e^{2\pi j(f_0+m\Delta f)t+\phi_k} \tag{1}$$

comprising M sinusoidal carriers with frequency spacing Δf and arbitrary phases ϕ_k. For simplicity we will consider only the case of baseband signal generation at the transmitter and a direct conversion (similarly baseband) receiver. Discrete Fourier transform processing may be used to implement this architecture as shown in Fig. 1. Let N be the number of time samples that are generated by the transmitter's IDFT and then repeatedly transmitted to form the continuous output waveform and f_s be the sampling rate.

As previously described, the basis of the multicarrier technique is the extraction of source to receiver distance from the frequency dependent phase offsets of the carriers. That is, if the estimate of the phase offset slope is such that the m_{th} carrier phase offset is $m\Delta\theta$, then the signal time delay is obviously

$$\tau = \frac{\Delta\theta}{2\pi\Delta f} \tag{2}$$

and the resulting distance estimate is $D = \tau c$ where c is the speed of light.

Equation (2) provides the fundamental relation between differential phase and delay, but it hides two important aspects of the indoor positioning problem: (1) the existence of multipath which renders the implication of the equation (that a given measured phase difference corresponds to a given *single* path delay) invalid and (2) the existence of noise which necessitates some mitigation technique for acceptable performance.

Both of these problems are addressed by the MC-UWB approach. As in similar systems, the underlying physical principal is that time delay is related to phase shift by $\Delta t = \Delta\phi/\omega_m$ for a given frequency ω_m. Consider initially a single unknown path length. Our system *samples* the phase shift of that path by using a large number of carriers so that for each value of delay corresponding to a single path, the phase sample values due to each carrier frequency take on a periodic form (since phase repeats each 2π radians). The period (or frequency) of this progression of phases maps to the unknown delay value for the particular path. For multiple paths we have a linear superposition of these signals, which may be resolved individually with a sufficient number of carriers. Also, if the number of carriers is larger than the minimum for our number of paths, we have an overdetermined system which may be solved by a technique such as singular value decomposition to separate signal effects from noise effects.

Hence, the heart of the algorithm is a means to accurately estimate a number (corresponding to the number of multipaths) of non-harmonically related *frequencies* in the presence of noise. These frequencies are uniquely related to the desired path delays (within the ambiguity range determined by the system signal parameters. Modern spectral analysis techniques are employed here to determine these frequencies. Note that Fourier analysis would not be optimal for this task because of the lack of harmonic spacing and the resulting poor noise performance.

Refer to the earlier papers [1-3] for a description of the means that are employed to eliminate absolute time references through time difference of arrival techniques and the means we have adopted for autocalibration and for solving for the 3D location fixes from such distance estimates.

CRAMER RAO BOUND FOR SINUSOIDAL FREQUENCY ESTIMATION

The problem of estimating the frequencies of multiple, arbitrarily spaced, sinusoidal signals from a noisy linear combination has a long history. Modern analytic and computational methods are now known collectively as Modern Spectral Analysis [4,5]. The current effort is based upon the application of the state space formulation of modern spectral analysis which achieves frequency estimation performance very near the Cramer Rao Bound (CRB).

Rao and Arun [6] provide an expression for the CRB for estimation of the phase step $\Delta\theta_k$, which can be expressed in terms of some of the variables defined above as

$$E\left\{\Delta\theta^2\right\} = \frac{6}{M^3}\frac{\sigma_n^2}{|c_1|^2}, \tag{3}$$

where σ_n is the standard deviation of the additive white Gaussian noise (AWGN) associated with each frequency sample and c_1 is the Fourier amplitude of M complex carriers comprising the received multicarrier signal. In [2] we showed that in terms of the total signal power, P_s, white noise power spectral density, N_0, number of real sinusoids, M, occupying a sub-band of width B Hz and sampled at rate $f_s = 2B$ for a time window of $T = \frac{N}{2B}$, the variance of the delay estimate becomes

$$\sigma_\tau^2 = \frac{3}{8}\,\frac{N_0}{\pi^2 B^2 T P_s}. \tag{4}$$

This level of performance has been confirmed both with signal processing simulations and with laboratory implementations of the system.

An important limitation of the above analysis and confirming experiments is that the model is based on a single source signal (no multipath) which generates a single sinusoidal phase pattern in the carrier complex amplitudes. Experimental results indicated a breakdown of this performance bound for two or more signal paths (hence, effectively, multiple sources) when their path length difference was less than a certain limiting value, the size of which was a function of the bandwidth of the full multicarrier signal. However, when multiple signals were spaced by even slightly greater amounts, the variance of distance estimates once again agreed with the above analysis.

In the next section we display an approximate analysis that serves to explain this performance breakdown and provides an estimate of the limiting spacing of sources below which the original single-source CRB fails.

MULTIPATH PERFORMANCE ANALYSIS

We begin our analysis by simplifying the form of the essential equations for the cases of two distinct paths and two carriers to obtain a general notion of why the solution for multiple paths may break down for sufficiently close paths and what the solution behavior is on break down. We will then develop a theory that describes the SNR at which failure to separate two nearby paths will occur.

Let $a\epsilon = 2\pi d_a\epsilon/c$ where $d_a\epsilon$ is the distance of source a from the receiver. Hence $d_a\epsilon/c$ is the time delay. Let a second source (representing a multipath signal) have effective distance $d_b\epsilon$ so $b = 2\pi d_b\epsilon/c$. Here, ϵ plays the role of a separation scaling factor. Suppose two carriers with a frequency difference of Δf are transmitted and received. To simplify the analysis we assume that phase references are chosen such that the complex amplitude of the signal at the higher frequency may be written as:

$$Ae^{ja\epsilon\Delta f} + Be^{jb\epsilon\Delta f} \tag{5}$$

where A and B represent the amplitudes of the two source components. Then, for small separations, this complex amplitude is given approximately by its Taylor series in epsilon, the first few terms of which are

$$(A+B) + (jAa + jBb)\,\Delta f\epsilon + \tag{6}$$

$$-\frac{1}{2}\left(Aa^2 + Bb^2\right)\Delta f^2\epsilon^2 + \cdots.$$

Note that if this signal is corrupted by noise and the factor $\epsilon^2 \Delta f^2$ is small, then there is no possibility of recovery of both a and b since they are linearly combined in the first order term. Thus we expect the ability of multipath resolution to break down at a given SNR whenever the separation-bandwidth product falls below approximately unity. At best in this case we can only treat the complex amplitude as one arising from a single source and the distance parameter that would be extracted from that first term would be $(Aa + Bb)/(A + B)$, that is, the single source would appear to be at a weighted average of the locations of the two sources. In the experimental results that follow we indeed see this form of weighted average combining of multipath distances for insufficient SNR conditions.

Unfortunately, the non-linearity of the solution for both a and b prevents ready analysis of the point at which the break down predicted above would occur with respect to SNR. Because of the nonlinearity, classical linear theoretical approaches such as the CRB cannot be applied. To obtain the minimum required SNR for resolution of two source paths as a function of their separation we will take an approach motivated by super-resolution imaging theory.

The problem of super-resolution imaging has had a long history in the optics and radar communities [7-10]. It is now well understood that a function of a finite width can be theoretically completely reconstructed, in all detail, from a strictly band limited representation of that function. This remarkable reconstruction is possible because the Fourier transform of a function with finite support is analytic, and, any analytic function is completely defined by any section of itself [7]. This reconstruction can be performed by a linear decomposition of the filtered function in terms of prolate spheroidal wave functions (PSWFs), normalization of the coefficients obtained by the eigenvalues of the (PSWFs) with respect to the filtering operation and then reconstruction using the same basis set.

However, it has also been known for quite some time that though theoretically possible, in practice the result provides little to no actual benefit for the general case [10-11]. The problem arises from the fact that the eigenvalues of the PSWFs diminish in size very quickly, approaching zero exponentially, as the attempted reconstruction resolution exceeds the classic time resolution limit of $1/2B$ (known variously as the Rayleigh, diffraction or Shannon limit) where B is the pass band width. These small eigenvalues lead to inversely large accentuation of any noise in the signal. Thus it is widely accepted that no practical increase in resolution can be obtained by this technique. However, as noted in [12] several studies have claimed significant improvements in radar imaging when the object being imaged has a simple structure consisting of point-like features and non-linear processing is employed. In the following we obtain a notion of why super-resolution is achievable in these cases while estimating the performance that may be extracted in our case with respect to the non-linear super-resolution extraction of the location of two point-like sources.

In the case of linear super-resolution processing, it has been shown [13] that the accentuation of noise from eigenvalue normalization prevents useful application, in the construction of the solution, of the k_{th} PSWF component where $k \geq n$, such that $\frac{1}{\lambda_n(w)} \geq \frac{S}{N}$, where the PSWF eigenvalues $\lambda_k(w)$ are monotonically decreasing functions of the component index, k, and functions of the parameter w which is determined by the finite extent of the original function and of the band limit of the given filtered representation. In effect, only n degrees of freedom may be usefully extracted from the given data. The parameter w is given by $w = \pi BT$ where B is the band limit is Hz and T is the time limit in seconds in the case of finite support time function reconstruction. Furthermore, $\lambda_k(w)$ is a monotonically increasing function of c. For small w, the following asymptotic expansion for $\lambda_k(w)$ can be found in [14] and was used to generate the results that follow.

$$\lambda_n(w) = \frac{2\ \left(2^{2\,n} n!^3\right)^2 w^{2\,n+1} e^{-\frac{(2\,n+1)w^2}{(2\,n-1)^2(2\,n+3)^2}}}{\pi\left((2\,n)!\,(2\,n+1)!\right)^2} \tag{7}$$

Now, the state space modern spectral analysis method we employ is a nonlinear, model-based system that finds the best line-spectral match to the given data for a fixed and given number of spectral lines. One can argue that given a signal consisting of two spectral lines, this technique implicitly seeks the solution to a problem that can be stated as an iterative process: Successively solve a linear super-resolution problem with progressively smaller assumed window of function support until a solution is found with two resolved spectral lines. Hence, unlike the linear super-resolution algorithm in which the parameter w is held fixed and n is chosen for favorable recovery of an image comprising n PSWF terms, this non-linear approach fixes n at 2 so as to simply split two lines while w is varied until this is accomplished.

Thus we argue that the non-linear super-resolution process may function beyond the level provided by the usual theory with minimum required SNR for successful reconstruction (in the two point source model being considered here) given to within a scale constant β by:

$$SNR(d) \ \geq \ \beta\,\lambda_2(\pi B 2d)^{-1} \tag{8}$$

where d is the time/distance separating the two points and these are resolved into two pulses in the time/spatial domain of width d supported on a region of size $T = 2d$. The unspecified scale constant β accounts for the specifics of the pre-detection filtering of the carriers and the threshold that is used to judge performance breakdown. What is remarkable is that this very approximate argument results in excellent agreement with the data obtained from the Monte Carlo performance analysis described below as is shown in Fig. 1.

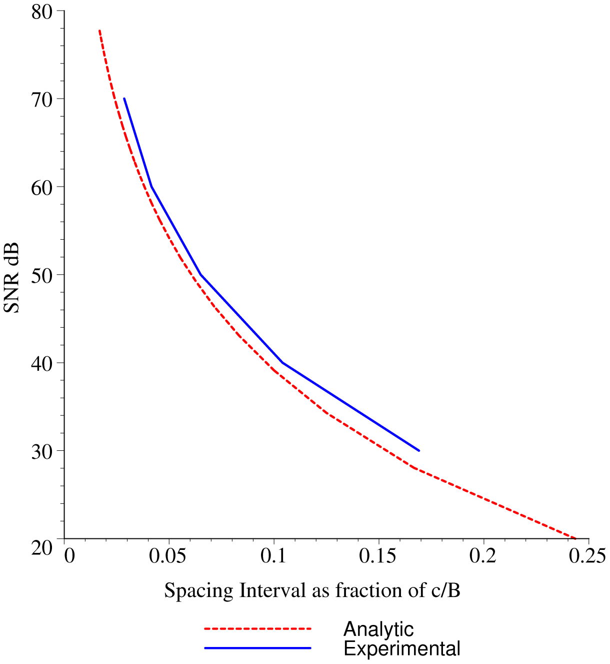

Figure 1: Comparison of approximate analytic analysis and experimental measurement of minimum required SNR for given path length difference.

SIMULATIONS AND LABORATORY TEST RESULTS

System performance is being determined via four approaches: (1) Analytical methods; (2) Monte Carlo simulations; (3) Baseband laboratory tests; (4) RF tests of the complete system. Previous papers and the results of the previous section of this paper have presented analytical results for the major aspects of system design, including: Cramer Rao bound for range (TOA/TDOA) accuracy in the absence of multipath; geometric positioning accuracy in the presence of ranging errors and reference node siting choices; and multipath resolution as a function of system bandwidth and signal to noise ratio.

Monte Carlo simulations provide a convenient and realistic environment for collection of detailed system performance data and for validation of analytical results. In these simulations all of the transmitted signal generation and received signal processing are accomplished with the same software as is used in the actual system. Only the signal transmission and noise injection aspects are simulated. Propgagation delay and multipath are simulated via simple delay and linear superposition of the carrier signals corresponding to the desired path lengths. Pseudorandom noise is then added to achieve the desired signal to noise ratio. This provides a controlled environment within which to determine system performance as a function of:

- Signal bandwidth,
- Number of carriers within the bandwidth,
- Number of multipath signals,
- Relative delay (path length difference) among the multipath signals,
- Signal to noise ratio,
- Internal model parameters of the frequency and position determination algorithms.

To date, system performance in the presence of two signals (direct and one indirect path)as a function of SNR and bandwidth has been extensively investigated as shown in Figures 2-5. These figures record estimation results for the direct path in the presence of one indirect path of the indicated additional path length with amplitude equal to one half of the direct path amplitude. System bandwidth is incremented between the figures. In each case the general performance characteristic is the same: for large

SNR and large difference in path length the correct result (8 m in this simulation) is obtained. For a given bandwidth and SNR, at some path length difference the distance estimate error increases rapidly with decreasing path length difference. After this "breakdown" point, individual paths are no longer resolvable; rather, a single path is identified whose estimated length is a weighted mean of the direct and indirect actual path lengths, weighted by the respective signal amplitudes. It should be noted that this result is well behaved, in that the error is bounded by the path lengths, and that as the path length difference decreases, the error also decreases.

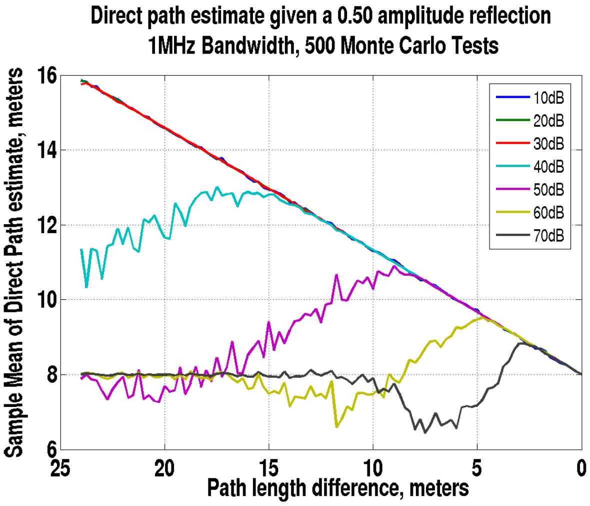

Figure 2: System performance for 1 MHz bandwidth, two paths with the indicated path length difference.

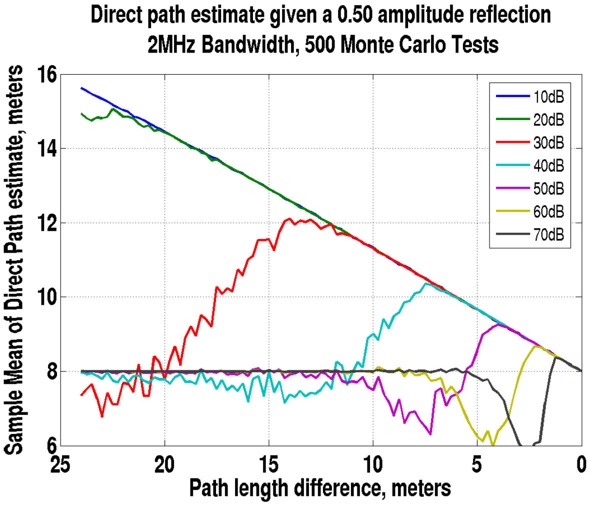

Figure 3: System performance for 2 MHz bandwidth, two paths with the indicated path length difference.

Figure 6 illustrates the effect of an increase in amplitude of the reflected path, to a value equal to the direct path amplitude. There is no change in system performance characteristics. The only difference is caused by the change in weighting factors for the non-resolved result after the resolution limit is exceeded.

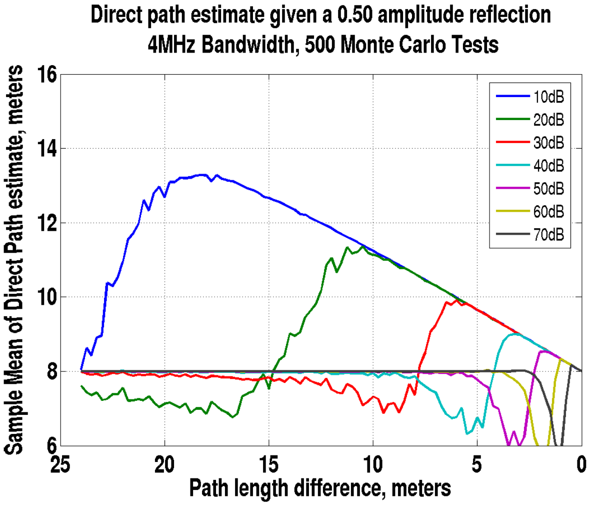

Figure 4: System performance for 4 MHz bandwidth, two paths with the indicated path length difference.

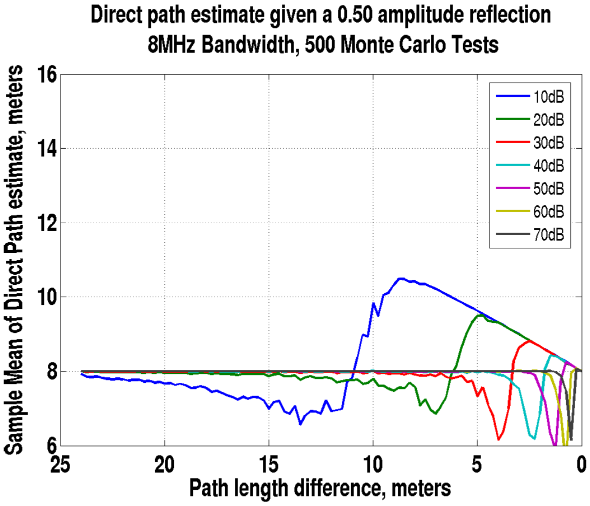

Figure 5: System performance for 8 MHz bandwidth, two paths with the indicated path length difference.

It is possible to display the estimates for both direct and reflected paths as in Figure 7. Prior to the breakdown distance the two results match the input signals: an 8 meter delay and a delay of $(8 + \Delta)$ meters. After breakdown only one path is resolvable and the result for the second requested path becomes a noise term.

The third approach to verifying results consists of laboratory tests making use of all hardware compo-

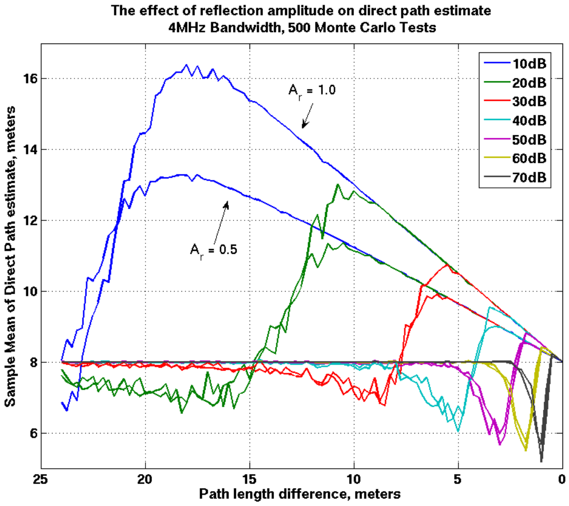

Figure 6: Illustration of the effect of change in amplitude of reflected path.

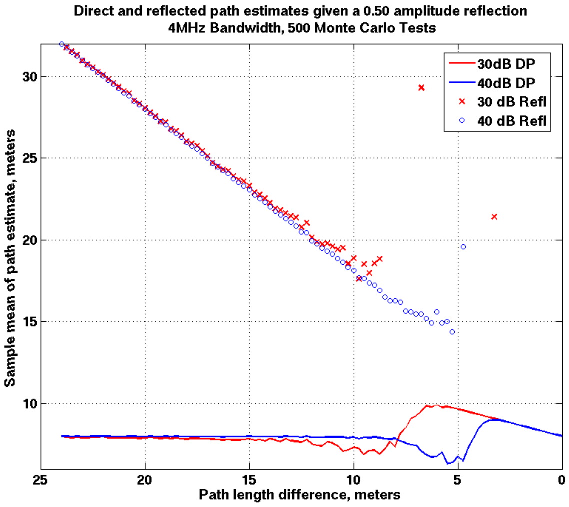

Figure 7: Estimates of path lengths of both the direct and indirect paths are plotted.

nents except the RF transmitter and receiver. In place of these components, the signal is transmitted through one or more coaxial cables of known lengths placed in parallel to simulate a direct and one or more indirect paths. The results of such a test are shown in Figure 8. Prior to breakdown the performance is essentially identical to that predicted by the simulator. Also, as the difference in path lengths nears zero, the estimated length follows the weighted average as expected. Behavior in the region around the breakdown point appears more complex than is predicted by the simulations. These effects are under investigation.

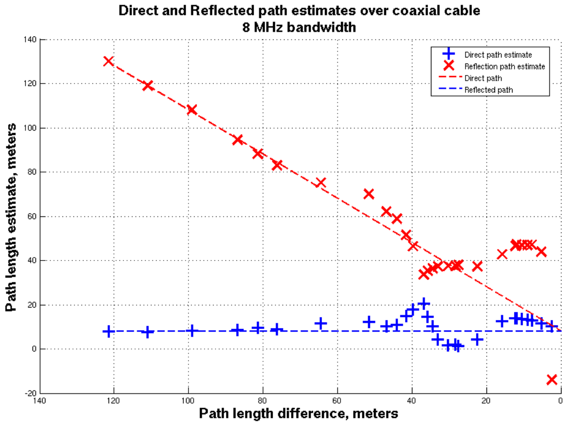

Figure 8: Laboratory results for two path lengths implemented with coaxial cables.

Finally, some initial RF tests have been conducted both outdoors and indoors with a very low bandwidth (4.5 MHz) system at a carrier frequency of 2.4 GHz. It was desired obtain baseline system performance in the absence of multipath so initial tests were conducted outdoors. Results are shown in Figure 9 for a range of transmitter-receiver spacings of 2 m to 24 m. Over this range the standard deviation of estimated ranges was approximately 0.3 m. An indoor test was conducted even though the system bandwidth is known to be insufficient to resolve the multipath signals that will be present. This result is shown in Figure 10 and demonstrates good performance at some ranges and large errors at others, presumably in cases with nonresolvable multipath at these frequencies.

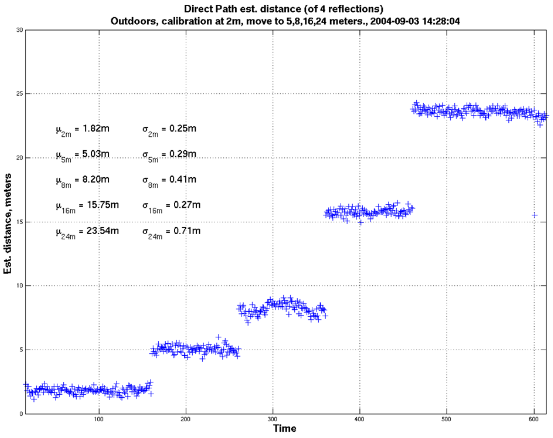

Figure 9: Results of outdoor ranging test.

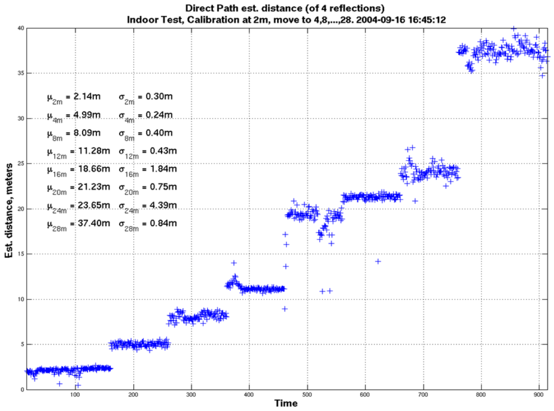

Figure 10: Results of indoor ranging test.

CONCLUSIONS

This paper has quantified the needed system bandwidth for a required range resolution as a function of signal to noise ratio. These results have been determined analytically and via simulation, and supported with initial laboratory tests. Next steps include the extension to wideband (50 MHZ) hardware and extensive testing of the position determination algorithm which makes use of the range information which was the subject of this paper.

ACKNOWLEDGMENT

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