

Threshold Setting and the Cycling of a Decision Threshold

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Abstract

When policy makers use a test result with a cutoff score in a decision, the cutoff threshold may change over time. An example is the threshold of "reasonable suspicion" used to justify a police search. Hammond (1996) postulated that a decision threshold will oscillate over time in response to competing pressures from affected constituencies, as unavoidable cases of false positives (e.g. innocent people searched) and false negatives (e.g. guilty people overlooked) emerge from the uncertainty of using an imperfect test (e.g. level of evidence) to predict the actual measure of interest (e.g. guilt). The structural underpinnings of a cycling threshold are analyzed in this theory-building article. First, we present a simplified converging model of Hammond's initial insight. Then, we present three alternative models: one with integral control representing the historical dissatisfaction of competing constituencies; a second model with delays in policy maker responsiveness; and a third with stakeholders' shifting constituencies.

KEYWORDS: decision, policy formulation, policy threshold, Taylor-Russell diagram, oscillation, police, search.

Introduction

To achieve perfect outcomes, policy makers ought to ground their decisions in perfect information. In practice, however, policy makers are faced not only with information fraught with uncertainty, but also of with the task of making standing policy for future decisions for which there is relatively little information at all.

In some situations, policy makers can make use of a scale or indicator score for which they select a cutoff point, treating future cases that fall above the cutoff point in one manner and cases that fall below the cutoff point in another. In each of these situations, a policy threshold must be selected to inform an important decision. Often there is no unique correct choice for the cutoff, but such a choice must be informed by social values and pressures based on uncertain information about consequences. This article presents a model for how policies might change over time for the special subset of policy decisions in which policy makers select a cutoff point on an indicator scale to inform their decisions.

Examples of this use of a cutoff point on an indicator include medical treatment decisions supporting or declining treatment according to a cutoff point on a medical test (Lohr, Eleazer, and Mauskopf 1998), applicant selection or rejection according to a cutoff point on a rating scale (Carlson 1967; Valenzi and Andrews 1973), a police officer's decision to search an individual or not based on a cutoff representing "reasonable suspicion" on a scale representing level of evidence (Ryan and Taylor 1988), and college decisions supporting or declining admissions based on a cutoff for a standardized test score (Dawes 1971).

As these thresholds have important consequences, it might be preferable if there were a uniquely defined formal solution that would minimize all errors. Nevertheless, the specific point selected is informed by current social values that might temporarily favor one or the other kind of error in the tradeoff. Policy makers are expected to make their cutoff selections while representing their constituencies, so that their provisional decisions meet the constituents' anticipated future preferences. In addition, policy makers

may be required to set this threshold in response to pressures from multiple constituents with opposing preferences.

This article consists of an exploratory investigation into theory building for the processes that affect the motion of policy thresholds under conditions of imperfect information. In particular, this article represents an investigation of the factors that lead to stable or cycling policy thresholds, using the system dynamics method to explore alternative models for these processes.

Characteristics of Threshold Setting Decisions

Four common features characterize the example situations described above and will be discussed in detail. This characterization is influenced by Egon Brunswik's lens model of judgment and Kenneth Hammond's application of the lens model to social policy (Brunswik 1956; Hammond 1996). First, a direct measurement of the distal event of interest is unavailable. Second, an indicator score is used in lieu of a direct measurement of the event. Third, the relationship between the local indicator and the distal event is statistically uncertain. Fourth, a value-based, rather than fact-based, threshold is used for a dichotomous decision.

Distal event. The event of interest is not directly knowable to the policy maker. In the case of a police officer deciding to initiate a search, the distal event is the true innocence or guilt of an individual's intentions or actions. For instance, the officer would ideally like to know whether the individual is planning to commit a crime, such as when an individual is casing a store for a future theft, or is concealing a current crime, such as the unlawful possession of a firearm. These are usually not directly measurable by a police officer before a search is initiated.

Proximal indicator. The policy maker makes use of available information to compute a score as a proxy for the event of interest. The officer must rely upon available cues to determine a score for perceived level of evidence. These cues might include the individual's appearance of nervousness, unwillingness to respond when addressed, and flight from the scene. While the officer may not be keeping a written quantitative score, it is assumed for modeling purposes that the officer has, at least implicitly, some kind of scoring system that represents his or her perceived level of evidence based on the available cues. The analysis of the components of this kind of score can be done using the judgment analysis technique (Hammond 1996; Cooksey 1996).

Statistical uncertainty. Most scores used as a proxy for a distal event of interest will not be perfect predictors. In other words, some officers may be better than others at judging when to initiate a search, but no officer's score for perceived level of evidence will be perfectly predictive of true innocence or guilt. Not only would a perfectly predictive scoring system require perfect judgment on the part of officers, but it would also require perpetrators and innocent people to send consistent signals to the officers on every occasion. Uncertainty is embedded in the judgment environment before the officer even arrives (Stewart, Roebber, and Bosart 1997).

Threshold for a dichotomous decision. The policy maker determines a cutoff point or threshold, and recommends one action if the score is above the threshold and another action if the score is below threshold (Swets 1992; Erev 1998). At some level of evidence representing reasonable suspicion, the police officer decides whether to search the individual or not. The police officer may require a lot of evidence before searching or very little. The cutoff for the officer's reasonable suspicion score is a value-laden choice

used to inform action. Policy makers have control over the location of these thresholds, but with that power comes the responsibility for the consequences of threshold-setting. In the next section, it will be described how these consequences cut two ways.

Uncertainty, Inevitable Error, Unavoidable Injustice

Hammond (1996) draws on signal detection theory (Green & Swets, 1966) to point out that any selected threshold automatically sets up the possibility of two complementary errors: false positives and false negatives. False positives occur when an indicator score above cutoff identifies a case as a "positive," when in fact the case was benign. In the reasonable suspicion example, a false positive consists of a search carried out on an innocent person. False negatives occur when an indicator score below cutoff identifies a case as a "negative," when in fact the case was not benign. In the police search example, a false negative consists of a decision not to search when, in fact, the individual was guilty.

Hammond transcends the debate of which error is the more egregious, focusing instead on how this "duality of error" is a necessary result of the use of a statistically uncertain test in a dichotomous decision. If a lower threshold for level of evidence is set for a police search, then violations of individuals' civil liberties result as innocent people are searched (false positives). If a higher threshold for level of evidence is set for a police search, then society bears the risk of guilty people going free (false negatives). Hammond refers to this duality of error as unavoidable injustice, made worse in proportion to the extent of uncertainty in the predictiveness of the indicator.

Hammond (1996, p.) makes use of a Taylor Russell diagram to illustrate the duality of error (see Figure 1). When points are plotted as individual cases, some of the

individuals with high level of evidence scores will be truly guilty. Their searches by officers represent the true positives in the upper right quadrant of the diagram. Some of the individuals with low suspicion scores will be truly innocent. These individuals who are not searched represent true negative cases and are depicted in the lower left quadrant.

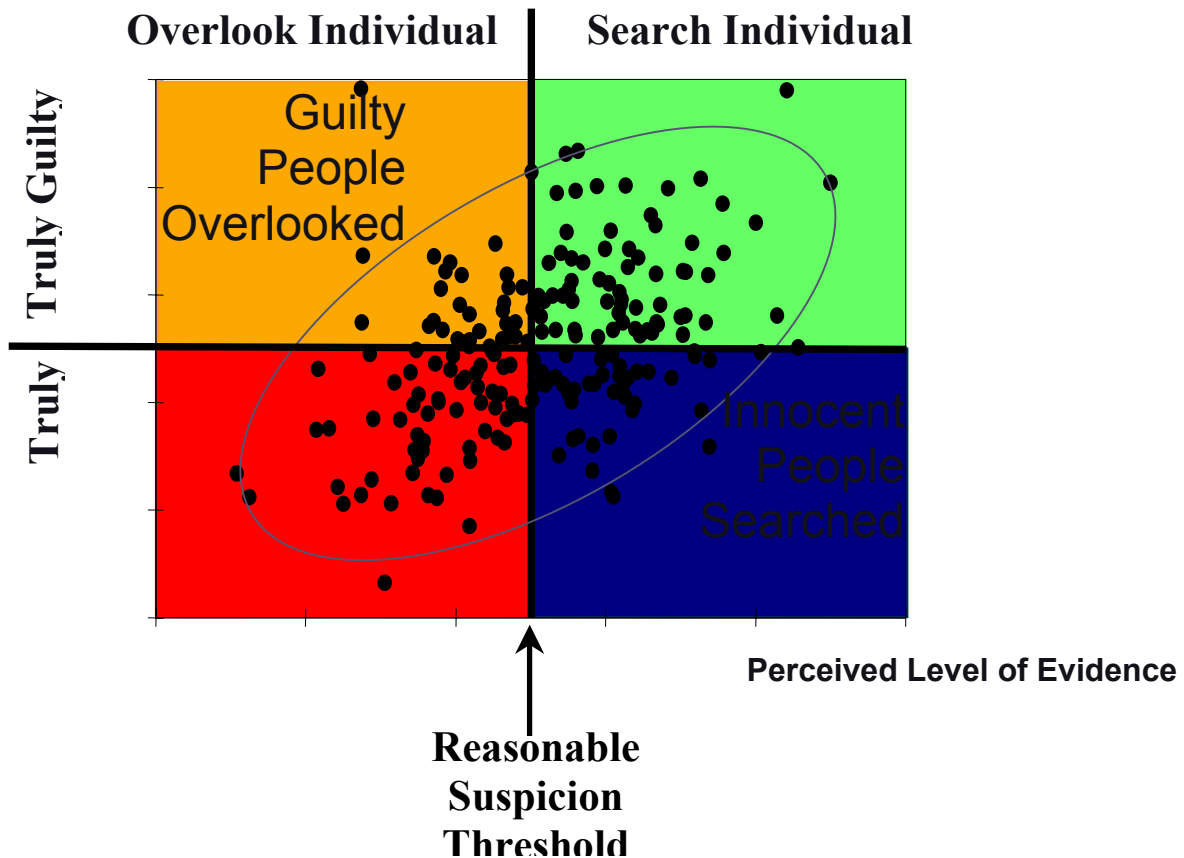


Figure 1. Taylor Russell Diagram.

However, in Figure 1, there are two other quadrants representing the duality of error: Those who are searched and innocent, representing false positives in the lower right quadrant; and those who are overlooked and guilty, representing false negatives in the upper left quadrant. The duality of error represents the fact that a change in the

decision threshold to solve one of these kinds of errors will result in more of the other kind occurring. See Figures 2 and 3.

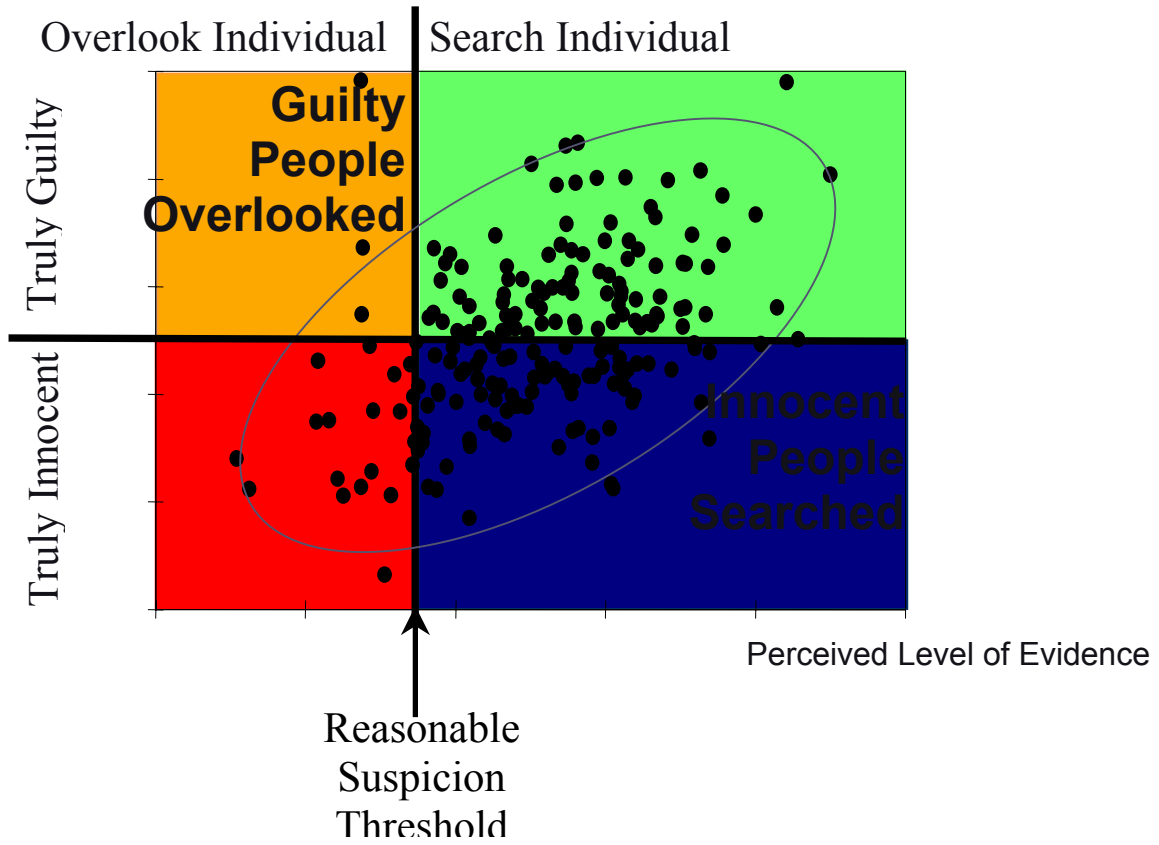


Figure 2. Taylor Russell Diagram with Low Threshold.

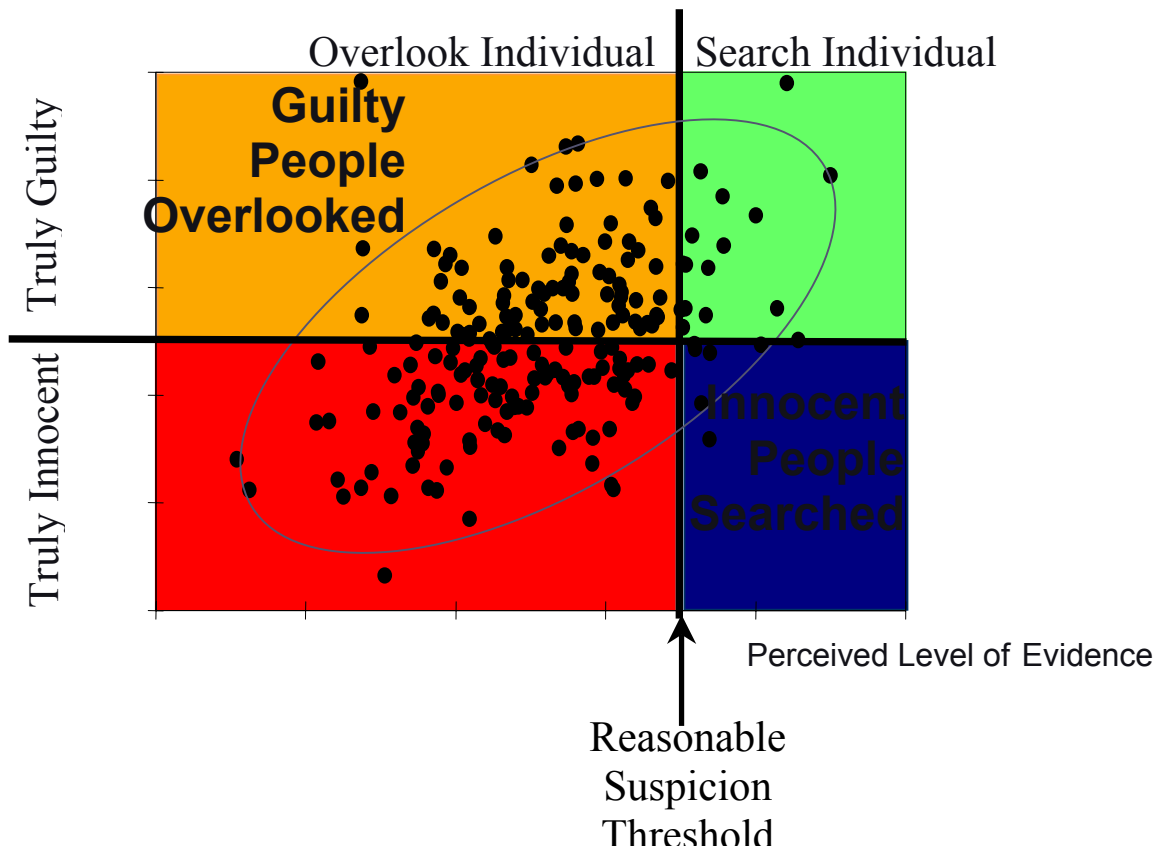


Figure 3. Taylor Russell Diagram with High Threshold.

Dynamic Hypothesis

In addition to using a Taylor Russell diagram to illustrate the duality of error, Hammond (1996, p. 55) puts forward a dynamic hypothesis about how a policy threshold will move over time. Hammond claims that as false positive and false negative errors occur, constituencies will develop to represent those treated unfairly, whether as the voice for the individual who loses his or her civil liberties or as the voice for the security-

conscious society that takes the risk when the guilty go free. These constituencies will lobby the policy makers to move the threshold to avoid the form of injustice on which they place their priority. Note that the two types of errors occur 180° out of phase with each other. To the extent that policy makers respond to the pressures and move the threshold, errors will begin to occur which represent the opposite form of injustice. Hammond claims that these opposing pressures on the policy maker will result in a cycling of the decision threshold over time.

Alternative Structures

This article constitutes a testing of Hammond's theory using system dynamics methods for an exploration of structures and parameters that could lead to policy threshold cycling.

Four different models are presented here for how the policy threshold could be affected by pressures from constituencies. The first, the *converging* model, is the simplest model that can be constructed from Hammond's theory. Hammond's description of the dynamics of threshold cycling suggests that opposing pressures from two constituencies, in response to false positive and false negative errors that are 180 degrees out of phase with one another, will yield an oscillation of the policy threshold over time. While this idea represents the insight for an oscillating model, it doesn't fulfill all the constraints required for cycling, such as negative feedback with time delays or more than one stock (Sterman 2000; Richardson and Pugh 1981). This converging model will be documented most fully, as the other models will consist of extensions of it.

The second is the *history* model. This model includes the assumption that constituents respond not only to current cases of false positives and false negatives, but also to cases from the past. In this model, constituents respond not only to current errors, but also respond to their memory for past errors when they determine how much pressure to exert on policy makers.

The third model is the *delay* model. This model incorporates the assumption that there is a delay between the time that policy makers exercise their decision and the moment that the new threshold is implemented in practice.

Finally, the fourth model is the *shifting constituency* model. This model incorporates the assumption that individuals may not remain within a constituency, but can become undecided or even switch to the opposing constituency after becoming concerned with the type of error that usually troubles the other constituency.

The models will begin in equilibrium and will be disturbed from equilibrium in either the *positive* or *negative* direction with a pulse input at the inflow to the decision threshold.

Modeling the Cycling of the Decision Threshold

Figure 4 represents a *converging* model based on Hammond's theory, using the example of the police search. The model could be generalized to any situation possessing the four characteristics mentioned earlier: a distal event (e.g., true guilt or innocence), a proximal indicator (e.g., perceived level of evidence), statistical uncertainty (e.g., an imperfect association between perceived evidence and true guilt or innocence) and a

decision threshold (e.g., "reasonable suspicion"), with competing constituencies pressing for opposite directions of change to that threshold.

In Figure 4, there are two balancing loops set in symmetrical opposition with parameters hidden for ease of exposition (see Appendix 3 for model equations). In the Civil Liberties Voice loop, a low reasonable suspicion threshold for level of evidence leads to more innocent people searched per year. When this searching of innocents exceeds an acceptable level of cases, the excess of unacceptable cases constitutes an affront to the civil liberties constituency. This dissatisfied constituency then exerts pressure through protests until policy makers decide to move the threshold higher. While this loop is a balancing one, which would tend to converge to a high threshold with no innocent people searched (e.g. no cases in the lower right quadrant of Figure 3 and therefore no further pressure), a comparable concern is set off in the security conscious constituency. The same increased reasonable suspicion threshold for level of evidence leads to a higher frequency of guilty people overlooked. Further criminal actions of the guilty lead to more cases than are acceptable, causing dissatisfaction in the security conscious constituency who then pressure the policy makers to lower the policy threshold. If the policy makers lower the threshold in response (as in Figure 2), they will then have to endure pressure from the civil liberties constituency.

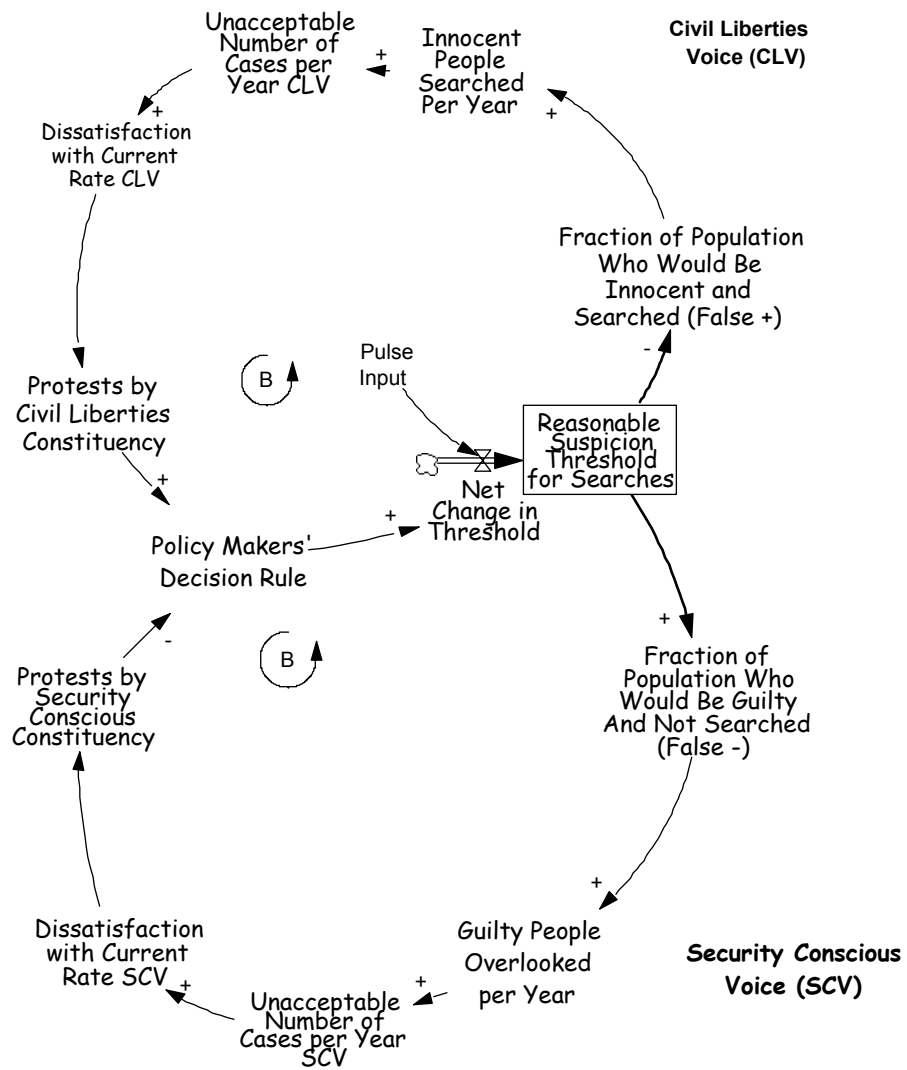


Figure 4. A representation of Hammond's theorized dynamics in the converging model.

Documentation of the Converging Model

The following are the specifics of the model described. Throughout all models presented in this article, the reasonable suspicion threshold for searches is characterized as a stock, with a net flow to the stock representing increases or decreases in the threshold. The value of a policy threshold determines the fraction of cases that are true positives, true negatives, false positives and false negatives as shown in Figures 1 to 3 above. For the models presented in this article, only false positives and false negatives drive pressure on policy makers, though it could be argued that a constituency for true negatives can drive policy as well, as demonstrated recently in protests over Mexico's college admissions process¹.

The fraction of false positive and false negative cases is calculated using a lookup function, derived using the Visual Basic for Excel macro listed in Appendix 2². This fraction of false positives or negatives is then multiplied by the total population considered for search to yield a number of false positives and false negatives in a given year. The total population considered was arbitrarily set at 100 people/year in this model.

Each constituency is assumed to tolerate a certain number of errors. To the extent that the number of false positives or negatives exceeds the tolerated number, there is dissatisfaction among the constituents. Currently the value of tolerated number was set arbitrarily to 5 people/year for both constituencies. The dissatisfaction resulting from

¹ The true negative constituency who were denied admission lobbied the government to lower the admissions standards. The government conceded, lowered the admission standards, and now many of these students are subject to attrition as they cannot fulfill their course requirements (personal communication, Ignacio Martinez and Luis Luna).

² The use of this macro assumes a bivariate standard normal distribution of points using two continuous measures (predicted and actual values), with a correlation of 0.6 between them, and a fixed vertical cutoff at the mean of the actual values. This macro yields the proportion of the "oval" or distribution of points in each quadrant for any threshold value, expressed in units of standard deviations from the mean, in the limit of an infinite population of cases.

each error is assumed to be 1 "dissatisfaction unit" representing an attitude survey score. This transformation value is arbitrarily defined for the sake of the model, and could be calibrated with measured survey scores, depending on the specific use of the model.

Constituency dissatisfaction is expected to yield protests or expressions of voiced concern over the errors resulting from the policy threshold. These could be newspaper letters to the editor, letters to political representatives, turnout at public protest rallies, or acts of protests such as strikes or embargos. The number of normal protests per year is arbitrarily set to 10, with a factor of 1 protest/unit dissatisfaction as the effect of dissatisfaction on voice.

The policy makers then receive voiced protests from both constituencies. Protests from the civil liberties constituency are greatest when the reasonable suspicion threshold is set at a low level of evidence resulting in more innocent people being searched. Protests from the security conscious constituency are greatest when the reasonable suspicion threshold is set at a high level of evidence, resulting in more guilty people overlooked and later crimes that could have been prevented.

The policy makers' decision rule assumes that policy makers take both voices into account and are pressed to move the threshold by the discrepancy between the pressures for changing the threshold. The actual calculation for the change is the increase resulting from pressure from the civil liberties constituency minus the decrease resulting from pressure from the security conscious constituency. The magnitude of change in each direction is the policy makers' responsiveness per protest (here set at 0.005 units/protest) multiplied by the number of protests. When that decision rule is divided by a threshold adjustment time set at 1 year, the net change in threshold is calculated.

Alternatively, the policy makers could use a biased rule that responds only to the louder of the two voices, but in that case, without any averaging over time, there would be a sawtooth shape to the threshold motion as policy makers abruptly shift from listening to one voice to listening to the other.

The models presented here begin in equilibrium, with a pulse disturbance applied to kick the system into motion. This pulse is a simple one-unit square-wave pulse with a duration of 10 time steps of 0.03125 years each, introduced at year 10. In practice, such a system would be jostled from equilibrium by random noise from the stochastic process that generates errors from a particular threshold value as depicted in Figures 1 to 3 above. As currently modeled, the equation used for errors is the expected value of the proportion, rather than a value including the standard error in the proportion, and a simple pulse is introduced at the threshold inflow for simplicity.

Test of the Converging Model

While Hammond claims that the threshold would cycle, this simple model of his theory converges, as it has no delays or additional stocks. In Figure 5, the model is tested with a positive or negative pulse disturbance fed into the threshold, and it is clear from the figure that the model converges from either side. The reason that the model converges to the same value from either the positive or negative direction is that the middle value is the point when the pressures from both constituencies perfectly balance, thereby negating each other with respect to their influence on the policy makers. This is the situation depicted in Figure 1, where each quadrant has an equal number of cases.

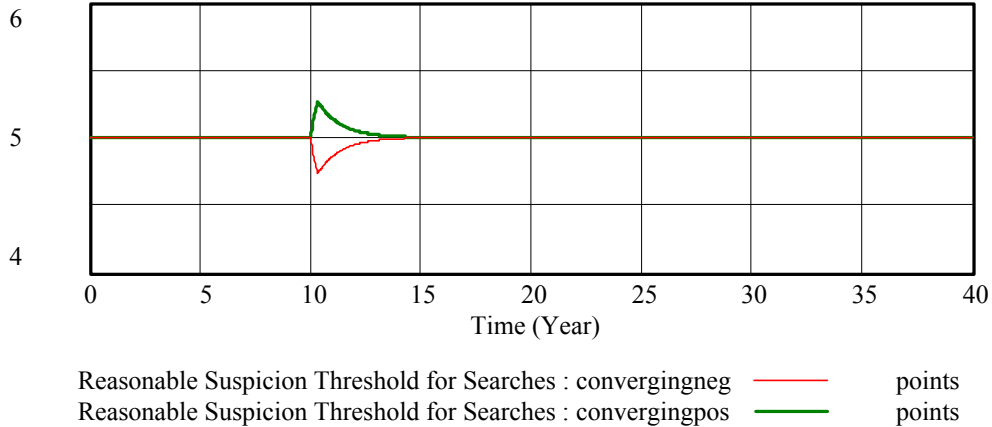


Figure 5. Convergence without cycling in the *converging* model.

Figure 6 depicts more of the dynamics of this model when it responds to a positive pulse disturbance. When the threshold increases, there are fewer protests by the civil liberties constituency, because fewer innocent people are searched. Nevertheless, more of the guilty are overlooked, due to the duality of error depicted in Figure 3. While the security conscious constituency is putting more pressure on the policy makers than the civil liberties constituency does, the policy makers respond by gradually lowering the threshold, until the false positives and false negatives are perfectly balanced. At this point, the system has been restored to equilibrium. A similar, but reversed, graph could have been generated for a negative pulse disturbance.

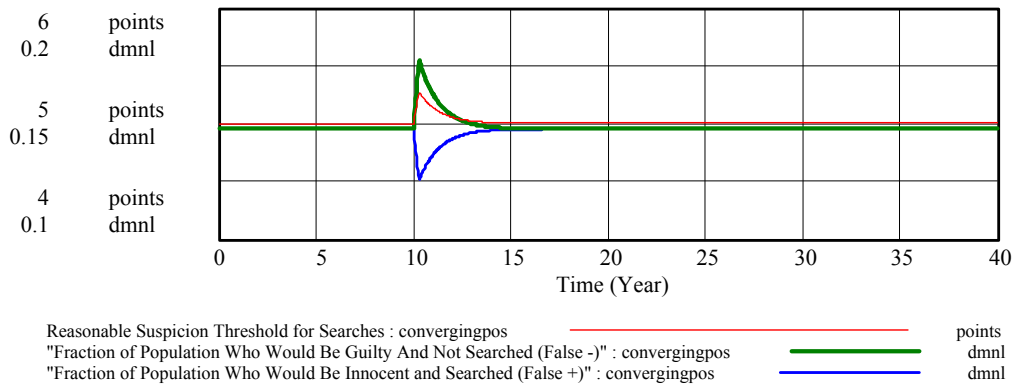


Figure 6. False Positives and Negatives and the Reasonable Suspicion Threshold.

Alternative models will now be presented that incorporate at least one extra stock, whether as history, delay or as shifting constituencies.

Expansion of the Model

History

It is plausible to assume that constituents respond not only to recent cases that represent the type of injustice of concern to them, but that they respond to their memory for past cases as well. The new case adds to a growing "grudge" against the policy and they speak out, weighting the historical cases more heavily than the current ones.

Figure 7 displays the model, extended to include this memory for past cases. Only the upper loop is shown, as it can be assumed that all changes are symmetrically introduced for both voices.

For this model, a certain proportion (arbitrarily set to 25%) of the unacceptable number of cases are retained in memory and then forgotten, with a forgetting time of 20

years. The remembered cases yield dissatisfaction that is weighted in with the recent dissatisfaction with the current rate of cases occurring, to yield a weighted dissatisfaction for each constituency. In this model, historic dissatisfaction is weighted at 90% while dissatisfaction with the current rate of occurrences is weighted at 10%. This weighted dissatisfaction is then expressed in protests.

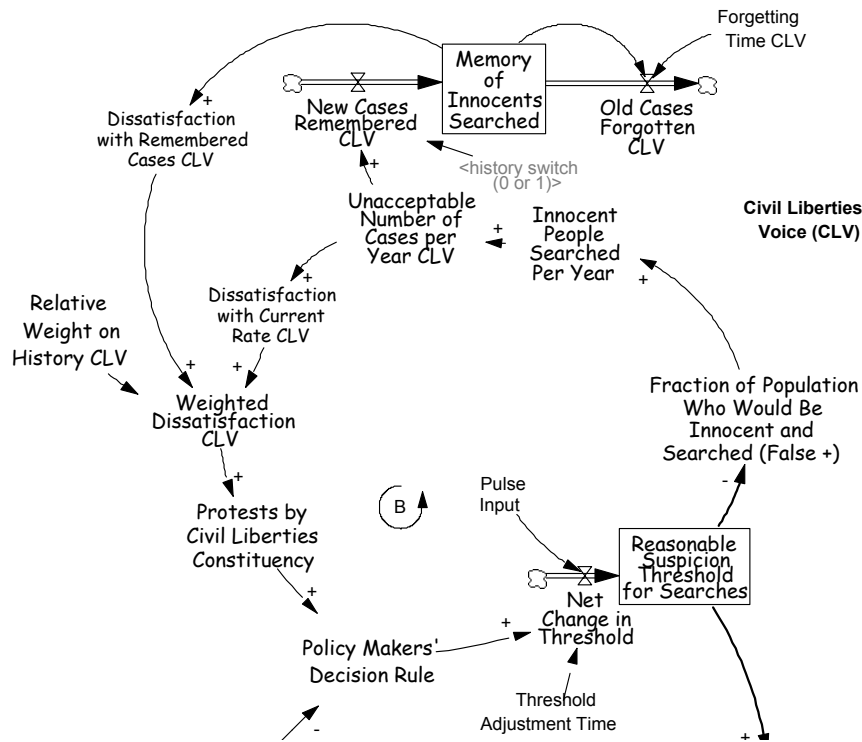


Figure 7. Model including history

Figure 8 shows the threshold and fractions of false positives and false negatives resulting from the threshold set. The pattern would be reversed for a negative pulse input. Note that the result of including the effect of history as an integral control is that the threshold decays in a similar way to the converging model, but this decay is modulated with the added oscillations.

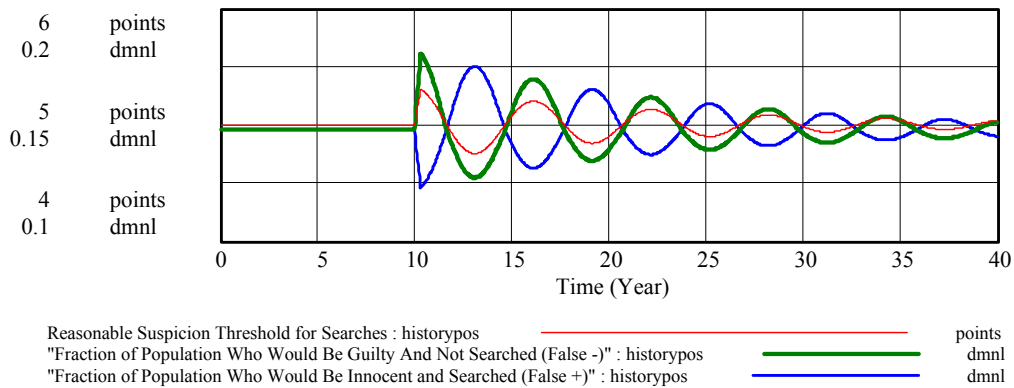


Figure 8. The damped oscillation of the threshold when history is included.

Delay

The introduction of a delay between policy formulation and implementation is the next possible way to model a cycling threshold. This delay is introduced after the policy decision rule and before the inflow to the threshold. Specifically, the delay is modeled as a smooth function between the policy makers' decision rule and the net change in threshold, using a threshold adjustment time of 25 years. A damped oscillation is the result of the added delay time as shown in Figure 9. The timing to convergence is slower for longer threshold adjustment times.

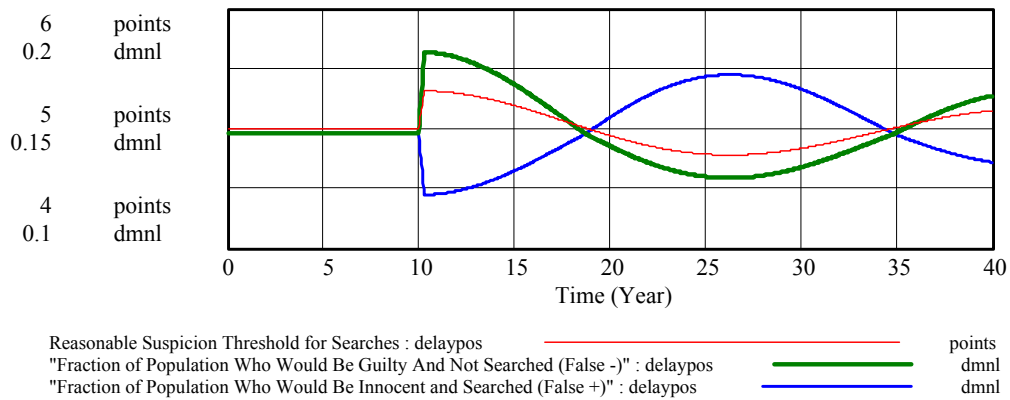


Figure 9. Reasonable Suspicion Threshold with Delay.

Shifting Constituencies

The final model to be considered here is one with shifting constituencies. An example would be a person from the civil liberties constituency becoming aware of guilty people going free, an error usually of concern to the security conscious constituency. The person then shifts constituencies, either to undecided or after a time to the security conscious constituency. Of course, the symmetrical situation could also be an example. As a result of the shift, the newly joined constituency gets larger, and increases its normal level of protests. See Figures 10 and 11 for the model.

In Figure 11, there are arbitrarily initially 1000 people in each stock, undecided, civil liberties and security conscious. In addition, there are 10000 in the population, so 7,000 apathetic people. When an unacceptable number of cases of one kind occurs, it could shock an open-minded member out of their singular voiced commitment. At that point, he or she becomes undecided and the fraction committed to the constituency

decreases. When that happens the normal level of protests is reduced shifting the pressure on the policy makers to the side of the constituents already objecting.

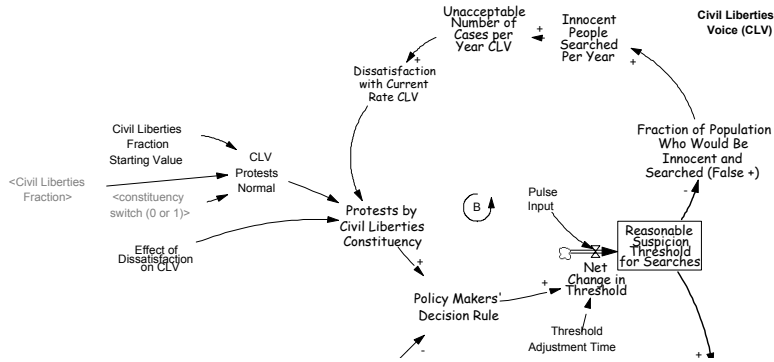


Figure 10. Model with shifting constituencies

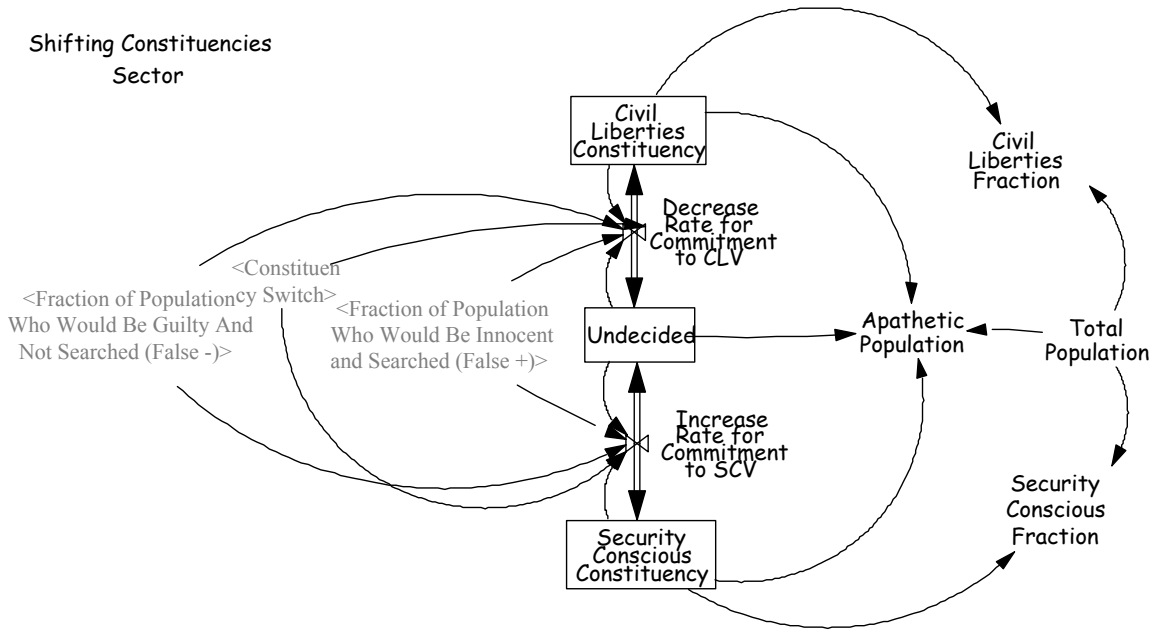


Figure 11. Shifting Constituencies Sector of Model

As shown in Figure 12, the effect of individuals shifting constituencies in the presence of an opposite error to the one usually of concern to them, is to lower the pressure from their initial constituency. So that initial constituency not only has a lack of recent errors, which lowers their voice but fewer voices calling for change in their direction. The result is no oscillation at all.

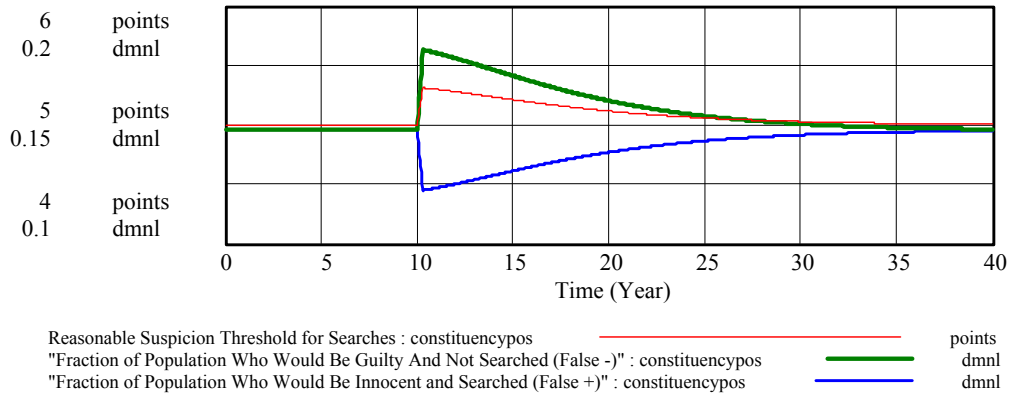


Figure 15. Shifting constituencies.

Conclusions

Four models have been presented, three of which could plausibly have led to oscillations. Not all parameters have been manipulated, not all possible alternative structures have been tested or proposed. The parameters were set arbitrarily to demonstrate the model. Nevertheless, these structures open the way to a future study of the motion of policy thresholds over time.

In addition, while oscillation was found in the history and delay models, these were also damped oscillations, at least for the parameters selected. It is plausible that in situations where errors occur randomly, the system would be regularly pushed out of equilibrium. Finally, the shifting constituencies model showed no oscillation as support dwindled from the countervailing side when errors occurred.

This article represents the beginning of a system dynamics approach to Hammond's posited cycling of the decision threshold in a policy decision. This theoretical model could provide insight into many policy situations, focusing attention on the trade-off between errors, rather than on the errors that have most recently occurred. The role of stakeholder pressures in threshold setting is a rich field worthy of more study.

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Appendix 1 The Excel Macro for Determining False Positives and False Negatives, given a threshold for the X and Y axis in a Bivariate Normal Distribution.

```
'  
' Calculation of Cumulative Bivariate Normal Distribution Macro  
' Using Z. Drezner Computation Method  
' Macro recorded 10/8/97 by Hwal W. Park  
'  
'
```

Option Base 1

Public Const Pi As Double = 3.14159265358979

Function formulae_1(a As Double, b As Double, rho As Double) As Double

```
Dim A_const(4) As Double  
Dim B_const(4) As Double  
Dim a_tick As Double  
Dim b_tick As Double  
Dim Double_sum As Double
```

```
Double_sum = 0  
A_const(1) = 0.325302999756919  
A_const(2) = 0.421107101852062  
A_const(3) = 0.13344250035752  
A_const(4) = 0.006374323486257  
B_const(1) = 0.133776446996068  
B_const(2) = 0.62432469018719  
B_const(3) = 1.34253782564499  
B_const(4) = 2.26266447701036
```

```
a_tick = a / Sqr(2 * (1 - rho ^ 2))  
b_tick = b / Sqr(2 * (1 - rho ^ 2))
```

```

For i = 1 To 4
  For j = 1 To 4
    Double_sum = Double_sum + (A_const(i) * A_const(j) _
      * (Exp(a_tick * (2 * B_const(i) - a_tick) _
        + b_tick * (2 * B_const(j) - b_tick) + 2 * rho * (B_const(i) _
          - a_tick) * (B_const(j) - b_tick))))
  Next j
Next i

formulae_1 = Sqr(1 - rho ^ 2) / Pi * Double_sum

End Function 'formulae_1(a,b,rho) for a<=0, b<=0, and rho<=0

Function mysgn(x) As Integer

If x >= 0 Then
  mysgn = 1
Else
  mysgn = -1
End If
End Function 'mysgn(x)

Function M(a1 As Double, b1 As Double, rho As Double) As Single

Dim rho_1 As Double
Dim rho_2 As Double
Dim sig As Double
Dim M1 As Double
Dim M2 As Double

If (rho >= 1) Then
  MsgBox "The value of the rho can not be greater than equal to 1. " & _
    "Please retry.", vbExclamation, "M:
Cumulative_Bivariate_Normal_Distribution "
  Exit Function
End If

If a1 <= 0 And b1 <= 0 And rho <= 0 Then
  M = formulae_1(a1, b1, rho)

```

```

ElseIf a1 <= 0 And b1 >= 0 And rho >= 0 Then
    M = Application.NormSDist(a1) - formulae_1(a1, (-1 * b1), (-1 * rho))
ElseIf a1 >= 0 And b1 <= 0 And rho >= 0 Then
    M = Application.NormSDist(b1) - formulae_1((-1 * a1), b1, (-1 * rho))
ElseIf a1 >= 0 And b1 >= 0 And rho <= 0 Then
    M = Application.NormSDist(a1) _
        + Application.NormSDist(b1) - 1 + formulae_1((-1 * a1), (-1 * b1), rho)
ElseIf mysgn(a1 * b1 * rho) > 0 Then
    rho_1 = (rho * a1 - b1) * mysgn(a1) / Sqr(a1 ^ 2 - (2 * rho * a1 * b1) + _
        b1 ^ 2)
    rho_2 = (rho * b1 - a1) * mysgn(b1) / Sqr(b1 ^ 2 - (2 * rho * a1 * b1) + _
        a1 ^ 2)
    sig = (1 - (mysgn(a1) * mysgn(b1))) / 4
    If a1 >= 0 And rho_1 >= 0 Then
        M1 = Application.NormSDist(0) - formulae_1((-1 * a1), 0, (-1 * rho_1))
    ElseIf a1 >= 0 And rho_1 <= 0 Then
        M1 = Application.NormSDist(a1) + Application.NormSDist(0) _
            - 1 + formulae_1((-1 * a1), (-1 * 0), rho_1)
    ElseIf a1 <= 0 And rho_1 >= 0 Then
        M1 = Application.NormSDist(a1) - formulae_1(a1, (-1 * 0), (-1 * rho_1))
    ElseIf a1 <= 0 And rho_1 <= 0 Then
        M1 = formulae_1(a1, 0, rho_1)
    End If
    If b1 >= 0 And rho_2 >= 0 Then
        M2 = Application.NormSDist(0) - formulae_1(0, (-1 * b1), (-1 * rho_2))
    ElseIf b1 >= 0 And rho_2 <= 0 Then
        M2 = Application.NormSDist(0) + Application.NormSDist(b1) _
            - 1 + formulae_1((-1 * 0), (-1 * b1), rho_2)
    ElseIf b1 <= 0 And rho_2 >= 0 Then
        M2 = Application.NormSDist(b1) - formulae_1((-1 * 0), b1, (-1 * rho_2))
    ElseIf b1 <= 0 And rho_2 <= 0 Then
        M2 = formulae_1(0, b1, rho_2)
    End If

    M = M1 + M2 - sig
End If
End Function 'M(a1,b1,rho)

```

Appendix 2 The Full Vensim Model Equations

Apathetic Population= Total Population-(Civil Liberties Constituency+Security Conscious Constituency +Undecided)
Units: people

Civil Liberties Constituency= INTEG (-Decrease Rate for Commitment to CLV, 1000)
Units: people

Civil Liberties Fraction= Civil Liberties Constituency/Total Population
Units: dmnl

Civil Liberties Fraction Starting Value= 0.2
Units: dmnl

CLV Protests Normal= IF THEN ELSE ("constituency switch (0 or 1)" = 1, Civil Liberties Fraction/Civil Liberties Fraction Starting Value,10)
Units: protests/Year

"constituency switch (0 or 1)"=1
Units: dmnl

Decrease Rate for Commitment to CLV="constituency switch (0 or 1)"*(("Fraction of Population Who Would Be Guilty And Not Searched (False -)")*Civil Liberties Constituency- ("Fraction of Population Who Would Be Innocent and Searched (False +)")*Undecided)/time to shift
Units: people/Year

"delay switch (0 or 1)"=0
Units: dmnl

Dissatisfaction per Current Person SCV=1
Units: dissatisfaction unit/(people/Year)

Dissatisfaction per current rate CLV=1
Units: dissatisfaction units/(people/Year)

Dissatisfaction per historical person CLV=5
Units: dissatisfaction unit/person

Dissatisfaction per Historical Person SCV=5
Units: dissatisfaction units/person

Dissatisfaction with Current Rate CLV=Unacceptable Number of Cases per Year
CLV*Dissatisfaction per current rate CLV
Units: dissatisfaction units

Dissatisfaction with Current Rate SCV=Unacceptable Number of Cases per Year
SCV*Dissatisfaction per Current Person SCV
Units: dissatisfaction units

Dissatisfaction with Remembered Cases CLV=Memory of Innocents
Searched*Dissatisfaction per historical person CLV
Units: dissatisfaction units

Dissatisfaction with Remembered Cases SCV=Memory of Guilty People
Overlooked*Dissatisfaction per Historical Person SCV
Units: dissatisfaction unit

Effect of Dissatisfaction on CLV=1
Units: protest/dissatisfaction unit

Effect of Dissatisfaction on SCV=1
Units: protest/(dissatisfaction unit*Year)

False Negative f([(0,0.000117367)-(10,0.493908)], (0,0.000117367), (1,0.00094349),
(2,0.00537066), (3,0.02196), (4,0.0657182), (5,0.147583), (6,0.257181), (7,0.363305),
(8,0.438563), (9,0.478193), (10,0.493908))
Units: dmn1

False Positive f([(0,0.000117361)-(10,0.493908)], (0,0.493908), (1,0.478193),
(2,0.438563), (3,0.363305), (4,0.257181), (5,0.147583), (6,0.0657182), (7,0.02196),
(8,0.00537065), (9,0.000943482), (10,0.000117361))
Units: dmn1

FINAL TIME = 100
Units: Year

Forgetting Time CLV=20
Units: years

Forgetting Time SCV=20
Units: years

"Fraction of Population Who Would Be Guilty And Not Searched (False -)"=
False Negative f(Reasonable Suspicion Threshold for Searches)
Units: dmn1

"Fraction of Population Who Would Be Innocent and Searched (False +)"=

False Positive f(Reasonable Suspicion Threshold for Searches)
Units: dmnl

Guilty People Overlooked per Year= "Fraction of Population Who Would Be Guilty And Not Searched (False -)"*Population Considered per Year
Units: people/Year

"history switch (0 or 1)"=0
Units: **undefined**

Increase Rate for Commitment to SCV="constituency switch (0 or 1)"*(("Fraction of Population Who Would Be Guilty And Not Searched (False -)"*Undecided - ("Fraction of Population Who Would Be Innocent and Searched (False +)"*Security Conscious Constituency)/time to shift
Units: people/Year

INITIAL TIME = 0
Units: Year

Innocent People Searched Per Year= "Fraction of Population Who Would Be Innocent and Searched (False +)"*Population Considered per Year
Units: people/Year

Memory of Guilty People Overlooked= INTEG (New Cases Remembered SCV-Old Cases Forgotten SCV,0)
Units: people

Memory of Innocents Searched= INTEG (New Cases Remembered CLV-Old Cases Forgotten CLV,0)
Units: people

Net Change in Threshold=("pulse switch (-1, 0 or 1)"*Pulse Input) + IF THEN ELSE("delay switch (0 or 1)"=1, SMOOTH(Policy Makers' Decision Rule, Threshold Adjustment Time), (Policy Makers' Decision Rule/Threshold Adjustment Time))
Units: points/Year

New Cases Remembered CLV= Unacceptable Number of Cases per Year CLV*Proportion Remembered CLV*"history switch (0 or 1)"
Units: people/Year

New Cases Remembered SCV=Unacceptable Number of Cases per Year SCV*Proportion Remembered SCV*"history switch (0 or 1)"
Units: people/Year

Old Cases Forgotten CLV=Memory of Innocents Searched/Forgetting Time CLV
Units: people/Year

Old Cases Forgotten SCV=Memory of Guilty People Overlooked/Forgetting Time SCV
Units: people/Year

Policy Makers' Decision Rule=Policy Makers' Responsiveness to Civil Liberties*Protests
by Civil Liberties Constituency-Policy Makers' Responsiveness to Security
Conscious*Protests by Security Conscious Constituency
Units: points/Year

Policy Makers' Responsiveness to Civil Liberties=0.005
Units: points/(protest/Year)

Policy Makers' Responsiveness to Security Conscious=0.005
Units: points/(protest/Year)

Population Considered per Year=100
Units: people/Year

Proportion Remembered CLV=0.25
Units: dmnl

Proportion Remembered SCV=0.25
Units: dmnl

Protests by Civil Liberties Constituency=CLV Protests Normal*Weighted Dissatisfaction
CLV*Effect of Dissatisfaction on CLV
Units: protests/Year

Protests by Security Conscious Constituency=SCV Protests Normal*Effect of
Dissatisfaction on SCV*Weighted Dissatisfaction SCV
Units: protests/Year

Pulse Input=(PULSE(10,10*TIME STEP))
Units: points/Year

"pulse switch (-1, 0 or 1)"=1
Units: dmnl

Reasonable Suspicion Threshold for Searches= INTEG (Net Change in Threshold,5)
Units: points

Relative Weight on History CLV=0.9*"history switch (0 or 1)"
Units: dmnl

Relative Weight on History SCV=0.9*"history switch (0 or 1)"
Units: dmnl

SAVEPER = TIME STEP

Units: Year

SCV Protests Normal=IF THEN ELSE("constituency switch (0 or 1)"=1, Security Conscious Fraction/Security Conscious Fraction Starting Value,10)

Units: protests/Year

Security Conscious Constituency= INTEG (Increase Rate for Commitment to SCV, 1000)

Units: **undefined**

Security Conscious Fraction=Security Conscious Constituency/Total Population

Units: dmnl

Security Conscious Fraction Starting Value=0.2

Units: dmnl

Threshold Adjustment Time=IF THEN ELSE ("delay switch (0 or 1)" = 0 ,1,25)

Units: years

TIME STEP = 0.03125

Units: Year

time to shift=0.5

Units: Year

Tolerated Number of Cases CLV=5

Units: people/Year

Tolerated Number of Cases per Year SCV=5

Units: people/Year

Total Population=10000

Units: people

Unacceptable Number of Cases per Year CLV=Max(Innocent People Searched Per Year-Tolerated Number of Cases CLV , 0)

Units: people/Year

Unacceptable Number of Cases per Year SCV=Max(Guilty People Overlooked per Year-Tolerated Number of Cases per Year SCV, 0)

Units: people/Year

Undecided= INTEG (Decrease Rate for Commitment to CLV-Increase Rate for Commitment to SCV, 1000)

Units: people/Year

Weighted Dissatisfaction CLV=Relative Weight on History CLV*Dissatisfaction with Remembered Cases CLV + (1- Relative Weight on History CLV)*Dissatisfaction with Current Rate CLV

Units: dissatisfaction units

Weighted Dissatisfaction SCV=Relative Weight on History SCV*Dissatisfaction with Remembered Cases SCV+ (1-Relative Weight on History SCV)*Dissatisfaction with Current Rate SCV

Units: dissatisfaction units