ABSTRACT Black Holes are precise mathematical solutions of the Einstein field equations of General Relativity. Some of the most exciting astrophysical objects in the Universe have been identified as corresponding to these mathematical Black Holes, but since no signals can escape their extreme gravitational pull, can we be sure that we have made the right identification?

I will show how the issue of reality of Black Holes can be addressed by nothing more than pen and paper. I will discuss three fundamental mathematical problems intimately connected to the issue of Reality of Black Holes: Rigidity, Stability and Collapse. I will then survey some of the main results which have been obtained in the last thirty years.