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HAROLD J GAY LECTURE SERIES

PDEs and Fractals



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From ODE to "beyond PDE"

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ABSTRACT In the late 1960s, Benoît Mandelbrot wrote that a piece of the coast of Brittany has Hausdorff dimension 1.6, but one difficulty with his statement is that there is no coast of Brittany! Indeed, there are tides and the boundary between the Atlantic Ocean and the land changes in a few hours, but the coast also erodes on a much longer time scale, and the rough nature of the coast which interested him took an extremely long time to appear. Using classical models in continuum mechanics leads to systems of PDE (partial differential equations), but modeling erosion involves some chemistry, and not so classical boundary conditions in some parts: even the movement of grains of sand resulting from the waves on a sandy beach is a much too hard mathematical question at the moment! In short, one has a complex evolution problem involving various length scales and time scales, and the mathematical tools of GTH (the general theory of homogenization) are not good enough yet for handling such questions, but it is already known that one should not expect an effective equation to be a system of PDE, since nonlocal effects appear in simpler problems: I coined the term "beyond PDE" for the new class of systems to consider, which is not clearly understood at the moment, in part because no good mathematical theory of nonlinear nonlocal effects exists.

In the mid-1980s, Alain LeMéhauté wrote that some electrodes develop a Hausdorff dimension between 2 and 3, adding that the Hausdorff dimension is different for old electrodes (he also pretended to invent fractional derivatives, which I had been taught by Laurent SCHWARTZ twenty years before); here, erosion involves electricity and chemistry coupled with continuum mechanics at various length scales and time scales, since electrons and ions moving near the electrode interact and create new chemical compounds, which must be evacuated for letting others approach the electrode.

Rough objects should not always be modeled by self-similar fractal sets, and something about the evolution processes which create the roughness should be studied in order to select better mathematical questions to consider. The creation of fjords in Norway or Greenland resulted from the erosion by glaciers (at a time when the level of the Atlantic ocean was much below its actual level), while the coast of Brittany resulted from the erosion by salted water, but the same word erosion corresponds to something more mechanical in the case of a glacier, and more chemical in the case of salted water: the two questions then correspond to two quite different evolution equations, creating different kinds of roughness at different scales.

Using ODE (ordinary differential equations) in mechanics corresponds to the 18th century point of view of classical mechanics, while the 19th century point of view of continuum mechanics uses PDE, but the 20th century point of view requires going "beyond PDE". In the absence of adapted mathematical tools, physicists sometimes guessed wrongly, and did not correct some obvious mistakes (like EINSTEIN creating a "fake Brownian motion" by confusing the jumps in position used by BACHELIER for modeling the stock market and the jumps in velocity observed by R. BROWN); correcting some less obvious mistakes made in the 19th and 20th centuries will require the work of 21st century mathematicians (with probably a good physical intuition) for improving the needed mathematical tools for describing the evolution of rough objects.

In the talk, I shall describe a few basic examples of the mathematical questions concerning effective equations for mixtures (i.e., roughness occurring in the bulk), and why nonlocal effects appear, without relying on probabilistic methods, which are not well adapted to the PDE of continuum mechanics or physics.

Geometry with its applications has been at the heart of the development of partial differential equations and boundary value problems since the very beginning. In physics, biology, economics, and other applied fields, a variety of new problems are now emerging that display unusual geometrical, analytical and scaling features, possibly of fractal type. The objective of these lectures is to acquire the view of outstanding mathematicians on the subject of differential equations and fractals, and their developments and applications, in a broad perspective encompassing both classical highlights and contemporary trends.

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Coffee and tea available before lecture time in Salisbury Labs 104

Participation of faculty and students is most welcome

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