WPI

HAROLD J GAY
LECTURE SERIES

PDEs and Fractals

Robert Hardt

W. L. Moody Professor of Mathematics
Rice University

Some New Uses of Functions with Finite Total Variation

Friday, December 9, 2011
11am, Bartlett Presentation Room

Abstract:

1) **Estimate the length of a continuous curve using drafting dividers.**

2) **Enhance an image by sharpening edges and smoothing roughness.**

3) **Find a spanning surface of least area in a very singular space X.**

These 3 problems can all be attacked using the notion of the Total Variation (TV) of a function.

For 1) the usual formula for the length of a curve $f : [a, b] \to \mathbb{R}^N$ is the definition of TV(f).

For 2) one considers a two-variable function $g : [a, b] \times [c, d] \to [0, 1]$ giving the grayscale intensity of an image in the rectangle $[a, b] \times [c, d]$. Assuming TV(g), suitably defined, is finite, edges and roughness are described using different parts of the derivative of g. The models for image enhancement that we will discuss involve interesting PDE's and many open questions.

For 3) we consider functions h whose values are finite sums of point masses in X. Assuming X has a distance function, we find a geometrically reasonable notion of the distance between 2 such sums. Then functions with TV(h) < ∞ essentially determine the surfaces and give a surprising amount of regularity. Analysis in singular spaces has had wide applications from algebraic geometry to data analysis.

Geometry with its applications has been at the heart of the development of partial differential equations and boundary value problems since the very beginning. In physics, biology, economics, and other applied fields, a variety of new problems are now emerging that display unusual geometrical, analytical and scaling features, possibly of fractal type. The objective of these lectures is to acquire the view of outstanding mathematicians on the subject of differential equations and fractals, and their developments and applications, in a broad perspective encompassing both classical highlights and contemporary trends.

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Coffee and tea available before lecture time in Stratton Hall 107

Participation of faculty and students is most welcome

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