



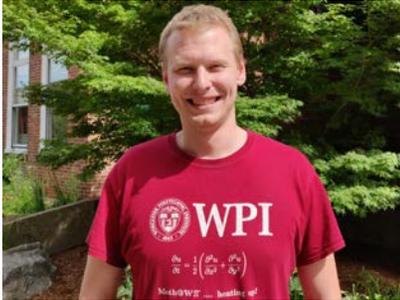
WPI

MATHEMATICAL SCIENCES

Brian Kodalen

PhD Candidate

Mathematical Sciences



September 26, 2018

Salisbury Labs, room 305

3:00PM

Dissertation Committee:

Dr. William Martin, WPI (Advisor)
Dr. Peter J. Cameron, Queen Mary,
University of London
Dr. Pádraig Ó Catháin, WPI
Dr. Peter Christopher, WPI
Dr. William M. Kantor, University of
Oregon
Dr. Gábor N. Sárközy, WPI

PhD Dissertation Proposal Presentation

Title: Cometric Association Schemes

Abstract: The combinatorial objects known as association schemes arise in group theory, extremal graph theory, coding theory, the design of experiments, and even quantum information theory. One may think of a d -class association scheme as a $d+1$ dimensional matrix algebra closed under the entrywise product containing I and J . In this context, an *imprimitive* scheme is one which admits a subalgebra (*subscheme*) of block matrices, also closed under the entrywise product. Such systems of imprimitivity provide us with *quotient schemes*, smaller association schemes which are often easier to understand, providing useful information about the structure of the larger scheme. One important property of any association scheme is that we may find a basis of $d + 1$ idempotent matrices for our algebra. A *cometric* association scheme is one whose idempotent basis may be ordered E_0, E_1, \dots, E_d so that there exist polynomials q_0, q_1, \dots, q_d with $q_i \circ (E_1) = E_i$ and $\deg(q_i) = i$ for each i . Imprimitive cometric schemes relate closely to spherical t -distance sets, sets of unit vectors with only t distinct inner products, such as equiangular lines and mutually unbiased bases. A similar type of association schemes known as *metric* schemes have been studied extensively with fundamental results such as a classification of imprimitive metric schemes dating back to the early 1970's. Analogous results for the cometric case weren't settled until nearly four decades later, with many other questions still open today.

After introducing association schemes with relevant terminology and definitions, this talk focuses on imprimitive cometric association schemes, especially those with small d . We will introduce and examine three projects spanning the previous four years and select theorems from each for the purpose of illustration:

- local connectivity of general association schemes;
- linked systems of symmetric designs;
- positive semidefinite cones of cometric association schemes.

In the first case we step towards answering a conjecture about the connectivity of association schemes which has been open for 20 years. In the second, we construct the only known examples not using the Kerdock set parameters. Finally, in the last project, we find parameter restrictions independent of those already known, resolving many open cases in online tables. We believe that these results, and the techniques used to reach them, will further our understanding of the young subject of cometric association schemes.