



WPI

MATHEMATICAL SCIENCES

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Tuesday, March 19, 2019

Campus Center
Taylor Conference Room
12:00PM

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PhD Dissertation Defense

Cometric Association Schemes

The combinatorial objects known as association schemes arise in group theory, extremal graph theory, coding theory, the design of experiments, and even quantum information theory. One may think of a d -class association scheme as a $d+1$ dimensional matrix algebra closed under the entrywise product. In this context, an imprimitive scheme is one which admits a subalgebra of block matrices, also closed under the entrywise product. Such systems of imprimitivity provide us with *quotient schemes*, smaller association schemes which are often easier to understand, providing useful information about the structure of the larger scheme.

One important property of any association scheme is that we may find a basis of $d+1$ idempotent matrices for our algebra. A *cometric* scheme is one whose idempotent basis may be ordered E_0, E_1, \dots, E_d so that there exist polynomials f_0, f_1, \dots, f_d with $f_i \circ (E_1) = E_i$ and $\deg(f_i) = i$ for each i . Imprimitive cometric schemes relate closely to spherical t -distance sets, sets of unit vectors with only t distinct angles, such as equiangular lines and mutually unbiased bases. Throughout this thesis we are primarily interested in three distinct goals: building new examples of cometric association schemes, drawing connections between cometric association schemes and other objects either combinatorial or geometric, and finding new realizability conditions on feasible parameter sets — using these conditions to rule out open parameter sets when possible.

After introducing association schemes with relevant terminology and definitions, this thesis focuses on a few recent results regarding cometric schemes with small d . We begin by examining the matrix algebra of any such scheme, first looking for low rank positive semidefinite matrices with few distinct entries and later establishing new conditions on realizable parameter sets. We then focus on certain imprimitive examples of both 3- and 4-class cometric association schemes, generating new examples of the former while building realizability conditions for both. In each case, we examine the related t -distance sets, giving conditions which work towards equivalence; in the case of 3-class Q -antipodal schemes, an equivalence is established. We conclude by partially extending a result of Brouwer and Koolen concerning the connectivity of graphs arising from metric association schemes.