WPI MATHEMATICAL SCIENCES

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PhD Candidate

Mathematical Sciences

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11:00AM-1:00PM

For Zoom Meeting info, please contact Rhonda Podell at rpodell@wpi.edu

Dissertation Committee:

Dr. Marcus Sarkis-Martins, WPI (Advisor)
Dr. Maksymilian Dryja, Warsaw University
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PhD Defense

Title: Non-Overlapping Spectral Additive Schwarz Methods

Abstract: Domain decomposition methods is one of the most important techniques commonly used in parallel computation for solving algebraic system of equations arising from the approximation of the partial differential equations. The basic idea is that instead of solving one huge problem in a global domain, it may be more convenient to solve smaller problems in subdomains simultaneously and combine their solutions to obtain an approximation for the global solution. This process is called "preconditioning the system" and is used as an iterative method and can be accelerated by Krylov space methods.

Additive Average Schwarz methods--AAS were designed as domain decomposition preconditioners for solving 2D and 3D elliptic problems with discontinuous diffusive coefficients across subdomains. These problems are then discretized using finite element methods and denote the element diameter by h and subdomain diameter by H. The Non-Overlapping Spectral Additive Schwarz Methods--NOSAS are the enhancement of AAS, which are robust for any type of coefficients.

NOSAS are two-level domain decomposition preconditioners that use non-overlapping subdomains, and the subdomain interaction is via the coarse space. The methods do not require a coarse triangulation, and the coarse problem can be seen as inverting a low-rank discrete harmonic extension on the subdomain interfaces. This rank is related to the lowest modes of a local generalized eigenvalue problem. Moreover, these methods can be modified to have a better localization and, therefore, better scalability. We prove that the condition number of NOSAS for heterogeneous elliptic problems does not depend on the coefficients, and with a specific choice of the threshold of the eigenvalues, the condition number is $O(H/h)$.

In this dissertation, we also consider NOSAS for Hybrid Discontinuous Galerkin discretizations of elliptic problems with heterogeneous coefficients and develop three-level domain decomposition preconditioners based on NOSAS. Finally, we show that NOSAS also works for Multiscale discretizations.