

## MA 2251 – Vector and Tensor Calculus – Term C23 Syllabus

**Instructor:** Prof. Guanying Peng

- Email: [gpeng@wpi.edu](mailto:gpeng@wpi.edu)
- Lectures: M-T-R-F 10:00-10:50 am, Salisbury Labs 104
- Office hours (Stratton Hall 105B): Mondays 11:00 am-12:00 pm, Thursdays 2:00-3:00 pm or by appointment

**TA:** Peiyao Lai

- Email: [plai@wpi.edu](mailto:plai@wpi.edu)
- Discussions: W 12:00-12:50 pm, Salisbury Labs 305
- Office hours: TBA

**Textbook (required):**

P. C. Matthews, *Vector Calculus*, Springer-Verlag, Berlin, 1998.

**Course description:**

Cat. I This course provides an introduction to tensor and vector calculus, an essential tool for applied mathematicians, scientists, and engineers. Topics covered include: scalar and vector functions and fields, tensors, basic differential operations for vectors and tensors, line and surface integrals, change of variable theorem in integration, integral theorems of vector and tensor calculus. The theory will be illustrated by applications to areas such as electrostatics, theory of heat, electromagnetics, elasticity and fluid mechanics. Recommended background: MA 1024.

**Course outline (*tentative, subject to change*):**

### I. Vectors

A. Basics (addition, negation, right-hand coordinates, length, unit vector)

B. Dot Product

1. definition vs.  $a_i b_i$
2. for perpendicular vectors
3. component of  $\mathbf{a}$  in the direction of  $\mathbf{b}$
4. equation of a plane

C. Cross Product

1. definition vs.  $\varepsilon_{ijk} a_j b_k$
2. for parallel vectors
3. right-hand rule
4. rotational velocity

D. Triple Products

### II. Integration

A. Basics (from Calculus II)

B. Line Integrals (work, circulation, conservative)

- C. Area Integrals (from Calculus IV), Surface Integrals (flux,  $\mathbf{n} dA = \underline{r}_x \times \underline{r}_y dydx$ )
- D. Volume Integrals (from Calculus IV; density)
- III. Differential Operators**
  - A. Background
    1.  $\text{Grad}(f(x,y))$  vs.  $\text{Grad}(F(x,y,z))$
    2. Directional Derivative
    3. Level Curve, Level Surface
  - B. Potential Functions
  - C. Divergence,  $\text{div}(\mathbf{u})$ 
    1. definition (interpretation) vs. formula
    2. solenoidal, divergence free
  - D. Laplacian
  - E. Curl,  $\text{curl}(\mathbf{u})$ 
    1. definition (interpretation) vs. formula
    2. irrotational, curl free
- IV. Suffix, Index, Tensor Notation**
  - A. Basic Rules
    1. Free Suffix
    2. Einstein Summation Convention
  - B. Special Tensors (Kronecker Delta, Alternating Tensor,  $\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$ )
  - C. Differential Operators in Index Notation (definitions, combinations, product rules)
- V. Green's Theorems (General Form)**
  - A. Divergence Theorem (Gauss's Theorem)
  - B. Stokes's Theorem (Curl Theorem); Green's Theorem
  - C. Related Theorems (Green's Identities)
- VI. Cartesian Tensors and Rotation**
  - A. Coordinate Transforms
  - B. Rotation Transform Rules for Tensors
  - C. Symmetric/Antisymmetric Tensors
  - D. Isotropic Tensors (*not on exam*)

**Grading:**

Homework	30%
Exam 1	35%
Exam 2	35%
Total	100%

Grades will be assigned as either A, B, C, I, or NR

- An average of 90% will ensure an A for the course.

- An average of 80% will ensure a B for the course.
- An average of 70% will ensure a C for the course.

**Homework:**

In order to encourage students to keep up with the course and to prepare for the exams, written homework problems will be assigned and collected each week. These will be graded for credit, and will constitute 30% of your grade. Your work should be neat and complete – and you will need to scan it to a *single* PDF and upload it to Canvas by the due date/time.

**Exams:**

There will be two in-class exams throughout the term. Each exam will constitute 35% of your grade. Exams are closed-book, no notes, no calculators.

*Exam 1:* Friday, February 3 (tentative), 10:00-10:50 am, Salisbury Labs 104

*Exam 2:* Friday, March 3 (tentative), 10:00-10:50 am, Salisbury Labs 104

**Missed Homework and Exams:**

In general, **no late homework submissions or make-up exams will be allowed.** Under unusual circumstances, a make-up or late submission may be allowed, however approval from the instructor must be obtained prior to (or within 24 hours if this is not possible) the deadline of an assignment or an exam. Approved make-up exams or late submissions of homework should be completed as soon as possible within a week of the original date. Rare and unusual circumstances, however, will be considered.

**Academic Accommodations Statement:**

Students with approved academic accommodations should plan to submit their accommodation letters through the [Office of Accessibility Services Student Portal](#). Should you have any questions about how accommodations can be implemented in this particular course, please contact your instructor as soon as possible. Students who are not currently registered with the Office of Accessibility Services (OAS) but who would like to find out more information about requesting accommodations, documentation guidelines, and what the accommodated interactive process entails should plan to contact OAS either by email [AccessibilityServices@wpi.edu](mailto:AccessibilityServices@wpi.edu), by phone (508) 831-4908, or by stopping by the office on the 5th floor of Unity Hall.

**Academic Dishonesty:**

There is no place for cheating in a university. As future leaders of our society, WPI students will be held to the highest ethical standards. Hard-working honest students can be assured that the instructor and the TA will do their best to preserve the integrity of their good work by being vigilant and promptly and forcefully prosecuting cases of academic dishonesty. Each student should be familiar with WPI's Academic Honesty Policy (<http://www.wpi.edu/Pubs/Policies/Honesty/policy.html>). If you are ever unsure as to whether your intended actions are considered academically honest or not, please contact your instructor in advance.

**Math Tutoring Center (NEW LOCATION: Gordon Library, Room 302):**

Schedule will be posted in Canvas as it becomes available.

**What to do if you get sick or if you are required to quarantine:**

- Stay in your residence and follow WPI protocol to report your symptoms and get medical advice.
- If you feel well, please contact your instructor to discuss how to keep up with the course remotely.
- If you are not able to keep up with the pace of class, try not to worry. Focus on rest and getting better. When you are ready to rejoin class, please contact your instructor by email and we will discuss how best for you to complete the course.