Exercise 1:
1. Show that if \( f_n \to f \) uniformly on \( \mathbb{R} \), each \( f_n \) is continuous, and \( x_n \to x \), then \( f_n(x_n) \to f(x) \).
2. Give an example of \( \{f_n\}, f, \{x_n\} \) satisfying the conditions of 1., except \( f_n \to f \) only pointwise, for which it is not true that \( f_n(x_n) \to f(x) \).

Exercise 2:
Let \((X, \mathcal{A})\) be a measure space and \( \alpha, \beta \), two finite measures on \( \mathcal{A} \). Let \( E \) be a subset of \( \mathcal{A} \) such that the smallest \( \sigma \)-algebra in \( \mathcal{P}(X) \) containing \( E \) is \( \mathcal{A} \). Assume that

(i). \( \alpha \leq \beta \),
(ii). for all \( E \) in \( \mathcal{E} \), \( \alpha(E) = \beta(E) \),
(iii). \( \alpha(X) = \beta(X) \).

Set \( \mathcal{T} = \{ T \in \mathcal{A} : \alpha(T) = \beta(T) \} \).

1. Let \( S, T \) be in \( \mathcal{T} \) such that \( S \cap T = \emptyset \). Show that \( S \cup T \) is in \( \mathcal{T} \).
2. Let \( T \) be in \( \mathcal{T} \) such that there are \( A_1, A_2 \) in \( \mathcal{A} \) satisfying \( T = A_1 \cup A_2 \) with \( A_1 \cap A_2 = \emptyset \). Show that \( A_1 \) and \( A_2 \) are in \( \mathcal{T} \).
3. Let \( S, T \) be in \( \mathcal{T} \). Show that \( T \setminus S, T \cap S, \) and \( T \cup S \) are in \( \mathcal{T} \).
4. Show that \( \mathcal{T} \) is a \( \sigma \)-algebra and infer that \( \alpha = \beta \).

Exercise 3:
Let the functions \( f_n \in L^p([0,1]) \) for some \( 1 < p < \infty \). Assume that \( \sup_n \|f_n\|_{L^p([0,1])} < \infty \).

1. Show that the sequence is equiintegrable, i.e., \( \forall \varepsilon > 0, \exists \delta > 0 \) such that for \( A \subset [0,1] \),
   \[
   m(A) < \delta \Rightarrow \int_A |f_n| < \varepsilon
   \]
   \( \forall n \in \mathbb{N} \). (Hint: write \( \int_A |f_n| = \int |f_n|\chi_A \), and use Hölder’s inequality).

2. Assume in addition that \( f_n \to f \) in measure on \([0,1]\) for some function \( f \), i.e., \( \forall \varepsilon > 0, \)
   \[
   \lim_{n \to \infty} m(\{ x : |f_n(x) - f(x)| \geq \varepsilon \}) = 0.
   \]
   Show that \( f_n \to f \) in \( L^1([0,1]) \), i.e., \( \lim_{n \to \infty} \|f_n - f\|_{L^1([0,1])} = 0 \).