Exercise 1:
Suppose that $f_n, f : \mathbb{R} \to \mathbb{R}$, and $f_n \to f$ uniformly. Show that
\[
\sup_{x \in \mathbb{R}} f_n(x) \to \sup_{x \in \mathbb{R}} f(x).
\]

Exercise 2:
Suppose $f_n, f \in L^1(\mathbb{R})$ with $f_n \geq 0$ and $f_n \to f$ a.e. Suppose also that
\[
m(\{ x \in \mathbb{R} : f_n(x) > f(x) + \alpha \}) \to 0
\]
for all $\alpha > 0$. Prove or give a counterexample that $\int f_n \to \int f$ must follow.

Exercise 3:
Let $E$ be a subset (not necessarily Lebesgue measurable) of $[0, 1]$.

1. Suppose the set $E$ has positive outer measure, i.e. $m^*(E) > 0$. Prove that for any $0 < \alpha < 1$, there exists an open interval $I \subset [0, 1]$ such that
\[
m^*(E \cap I) \geq \alpha m(I).
\]

2. Suppose $E$ is Lebesgue measurable, and there exists $0 < \alpha_0 < 1$ such that for any nonempty open interval $I \subset [0, 1]$, we have
\[
m(E \cap I) > \alpha_0 m(I).
\]
Prove that $m(E) = 1$. (You may use the conclusion of part 1 even if you cannot prove it.)