Exercise 1:
Consider the sequence of functions $f_n : [0, 1] \to \mathbb{R}$ where $f_1(x) = \sin\left(\frac{\pi}{2}x\right)$, $f_2(x) = \sin\left(\frac{\pi}{2} \sin\left(\frac{\pi}{2}x\right)\right)$, $f_3(x) = \sin\left(\frac{\pi}{2} \sin\left(\frac{\pi}{2} \sin\left(\frac{\pi}{2}x\right)\right)\right)$, and so forth. Thus $f_{n+1}(x) = \sin\left(\frac{\pi}{2} f_n(x)\right)$.

1. Does this sequence converge pointwise on $[0, 1]$? Prove or disprove pointwise convergence, and if the sequence converges, find the limit function $f$.

2. Does this sequence converge uniformly to $f$ on $[0, 1]$? Prove or disprove uniform convergence.

Exercise 2:
Suppose that $f \in L^1([0, \infty))$. Let $f_n : [0, 1] \to \mathbb{R}$ be defined by $f_n(x) = f(x + n)$.

1. Show that $f_n \to 0$ in $L^1([0, 1])$.

2. Show that for almost every $x \in [0, 1]$, we have $\lim_{n \to \infty} f_n(x) = 0$.

Exercise 3:
Suppose $f, g \in L^\infty(0, 1)$ with $f > 0$ and $g > f$. Prove or disprove that we must have $\|g\|_\infty > \|f\|_\infty$. 