MA 3257C SPRING 2025
NUMERICAL METHODS
FOR LINEAR AND NONLINEAR SYSTEMS
COURSE SYLLABUS

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Contents

1. Contact Information

Name	Email	Office Hours	Section	Lec/Dis Time	Location
Dr. Binan Gu	bgu@wpi.edu	M11-12, R1-2, F9-10 SL405A	C03	MTRF 8:00 - 8:50 am	HL 154
Taorui Wang	twang13@wpi.edu	T 2-4 pm SL 412	C03	W 8:00 - 8:50 am	HL 154

2. Course Description

This course provides an introduction to modern computational methods for the numerical solution of systems of linear and nonlinear equations, including systems, and their applications. Topics covered include solution of nonlinear scalar equations, direct and iterative algorithms for the solution of systems of linear equations, solution of nonlinear systems, and the eigenvalue problem for matrices. Both theoretical understanding of methods and their practical performance will be stressed. Computation is an essential component of the course, and there will be frequent computing assignments.

Recommended background: MA 2071 and some prior programming experience. Familiarity with MATLAB will be helpful but is not essential.

3. Textbook

Main textbook: *Numerical Linear Algebra* by Folkmar Bornemann, freely available on Springer under WPI wireless network.

The text is intended to serve as a background reference on most but by no means all of the material covered in class. **The material covered in the lectures constitutes the course content for which you will be responsible.**

Supplementary textbook:

(1) *Linear Algebra and Learning from Data (2019)* by Gilbert Strang, available here.
(2) *Numerical Analysis (2010)* by Richard Burden and Douglas Faires, Edition 9.

4. Course Objectives

4.1. **Goal for Each Topic.** For each topic, your main job is to understand

- Why is this topic introduced and studied?
- What are its fundamental connections to prior knowledge in this area?
- Learn the chronology of the topics introduced in this class, and understand the logic from one topic to the next.

4.2. **Main Topics.** This course explores the use of algorithms to solve systems of linear and nonlinear equations. The chapter and section numbers all refer to the textbook by Bornemann. The technical contents include (optional topics in parentheses):

- Error in numerical methods – Chapter 3 (Sections 10-13);
- Direct methods for linear systems – Chapter 2 (Sections 7-9);
- Eigenvalue approximation – Chapter 5 (Sections 18-21)
- Iterative methods for linear systems – my notes.
- Iterative methods for nonlinear systems – my notes.

Meanwhile, the students will develop their scientific writing skills and intuition, including

- Clear and concise use of the **mathematical** language.
- Responsible coding etiquettes, such as commenting and prefacing.
- Concept of accuracy, speed and stability of algorithms, and the ways to show/prove them.

4.3. **Goals.**

4.3.1. *Course Goals.* Two end goals are measured equally in this class:

(1) (Understanding) The ability to recall definitions, provide motivating examples and argue with mathematical logic.
(2) (Mechanical) The ability to carry out a few steps of important algorithms.

4.3.2. *Personal Goals.* There are a few personal goals I wish everyone achieves by the end of the class.

(1) **Learn how to learn**. This includes
   (a) Forming good learning habits:
      (i) Consistent contact with the course material, i.e., study a little every day.
      (ii) Work on one problem a day.
      (iii) Read for 10 minutes ahead of the lecture.
      (iv) Read for 10 minutes after the lecture.
   (b) Eradicating bad learning habits:
      (i) Give up too quickly before using **all** available (permitted) resources, i.e., lecture notes, examples, office hours and your brain.
      (ii) Immediately look for solutions online.
      (iii) Procrastinate until the due date.
      (iv) Cram before the exam.

(2) Learn about yourself: what are your strengths and weaknesses, academically, and mentally, before, during, and after this class?

5. Course Organization

The class typically meets 5 times per week: 4 lectures on MTRF, and 1 discussion section on W. Students are responsible for all material presented in lecture and discussion. My expectations for these activities are:

5.1. **Lectures:** The 200 quality minutes we spend together every week in lecture are meant to **introduce** the subjects. Students are expected to **know of** (i.e. *Lehr*) the introduced topics after every lecture. **Mastery** comes later after exercises. Lecture videos are captured by Echo360 and can be accessed through the course Canvas page.

There will be complementary videos on worked examples posted every week. Students are responsible for learning these examples in depth.

5.2. **Practice Problems:** The majority of your association with this course lies in **problem solving outside the classroom**. Students are expected to take the intuitions gained from lectures and put them into careful examinations by doing suggested practice problems (i.e. *Kunst*). Solutions to these problems are neither collected nor graded, but they form the main topics of discussion during the Discussion section. The problem selection is a combination of examples of the types of problems to be found on the exams, along with problems that are more detailed and which enhance learning the topics at a deeper level.

Extra credit problems may be assigned during lecture time. They normally involve filling out the details of a derivation and/or more challenging problems within the context of the course. Grades earned from these problems will be complemented to your overall HW grade (see below), though they will not help one exceed the maximum number of points possible.

5.3. **Discussion:** Students have an opportunity to sit down and work through the practice problems on a topic with an instructor and have their individual questions addressed. It is through the Discussion that the student can get major questions asked so that after Discussion, the student can continue to work through the practice problems to develop mastery.

In each Discussion session, everyone will solve some of the practice problems posted under Discussion on a clean piece of paper, to be submitted at the end of the Discussion. You will be graded by your attendance and performance in class, each out of 2 point. Discussion performance includes asking good questions, helping your classmates and completing more than 75% of the assigned problem numbers. These points will be added to your HW grade with a 1:1 conversion rate (without exceeding the maximum HW points – see non-exceeding bonus points below).

5.4. **Canvas:** Course materials can be found on the Canvas page. The material is organized by the section number of the topic in the text. In the event that lectures need to be delivered remotely, they will take place through Zoom.

Students are expected to spend an additional **10-14 hours per week** studying outside of class. This includes reading the text, organizing notes, and solving problems.

5.5. **Communications:** The primary interface for communication with the instructor & course staff will be email, the Canvas course website, office hours, and discussions. All information about the course will be maintained on the course web page in WPI's Canvas system. Check it often.

Check your WPI email daily. Students can expect a response to email within 24 hours on weekdays and within 48 hours on weekends.

6. Assignments

There are three different types of assignments for this course.

6.1. **Written Homework.**

Written Assignments weigh **considerably** in this course (see Grades). Take them seriously. These assignments involve handwritten solutions (electronic handwriting, using Goodnotes or similar apps, is allowed and even prefered) to mathematical problems from the course textbook and my lecture notes. Written solutions should be **second drafts** and thoroughly demonstrate your thought process, including justifications of steps. These assignments are due once a week, containing approximately **four lectures** worth of materials. They can be accessed through the Assignments Module on Canvas.

6.1.1. *Collaborations and References.* Collaborations among classmates are welcome. Every HW submission must declare a list of collaborators at the beginning, though each student is expected to compose his/her own solutions. **Extensive similarities between assignments without collaboration declarations will be considered as plagiarism and will be penalized severely.**

If the solution depends on online sources or textbooks, the student is responsible for citing them with a reference style of your choosing. Uncited solutions, if confirmed, will be considered as plagiarism and will not receive any credit. Please review AI Policy at the end of the syllabus.

6.1.2. *Return-and-correct Policy.* Each student has 1 day to correct his/her solutions after HW is returned. Resubmissions may be graded without penalty if the initial submission has sufficient effort in solving **every** problem. Poor attempts at the initial submission will receive little credit and no resubmission opportunities.

Every resubmission shall contain the original copy or state clearly which problems you are correcting. You must address every query for the correction to be considered valid, e.g., it is not allowed to just correct one out of two mistakes. The corrections should be written using a different colored pen/pencil, either in a side column clearly marked for short corrections, or on new sheets of paper for long corrections.

6.1.3. *File Type.* The initial submission and later corrections are to be submitted as **a single PDF file** via the canvas assignment module.

6.1.4. *Grading Rubric.* For each HW, we select four problems and grade each out of the four equally important points. We will comment on which criterion is missing from your assignment, to which you must pay more attention when revising.

Type	Submission Qualities
Presentation	Quality is neat and easily readable. Ample spacing between each problem.
Completion	Every problem has a committed attempt.
Syntax	Correct use of basic mathematical symbols: $=$ (equal to), $\Longrightarrow$ (implies), lim (limit sign before evaluation), $\to$ (converges), $\int f(x)\,dx$ (correct variable), etc.
Correctness	Completely correct, clear, & thorough write-up of problem solution, citing appropriate rules & theorems where appropriate.

6.1.5. *Completion Policy.* Each HW is out of 20 points, 16 of which go to the four graded problems. The remaining 4 points account for the completion of the ungraded problems. This completion is graded via an all-or-nothing scheme, i.e., if you miss one problem, you earn 0 points. You must make up for the solutions of the missed problems in revisions for the revision to be considered valid and earn at most 1 out of the 4 (completion) points back, should the resubmitted work be satisfactory.

6.1.6. *Poor work policy.* The instructor and the TA reserve the rights to reject HW with poor presentation and careless completion without further review of its content. The first rejection acts as a warning, and its resubmission incurs a penalty of 15% of total points; any HW rejection after the first offense automatically receives a score of zero.

6.1.7. *Late Work Policy.* Any late HW abides by a "15% of total points off per day late submission policy" unless a valid excuse is supported with proper documentation.

6.1.8. *Bonus Points.*

- Exceeding bonus points
  This type of bonus points includes extra credit problems assigned in class, bonus problems in (some) quizzes and (some) HW sets. They may help you achieve more than 100% in the assignments containing them.
- Non-exceeding bonus points
  This type of bonus points includes HW0 and Discussion Grades. They will only help you make up for the points you missed from HW assignments (only) and will not exceed the maximum points alloted for the assignment. For example, if you have scored 110/140 for all 7 HWs, and 15/20 for HW0 and 7/7 for the 7 Discussions, then your HW grade is (110+15+7)/140 = 132/140.

6.2. **Exams.** Two exams: 2/13 (Thursday), and 3/7 (Friday).

6.2.1. *Makeup Policy.* Unless there is extenuating circumstances, no makeup quizzes/exams are allowed. Students requesting quizzes/exams to be taken at times other than the proposed dates must provide valid reasons with proper documentation (medical, familial, or religious reasons), or notices at least a week in advance. A temporary grade of Incomplete is assigned until all makeup exams are finished (typically given during the first week of B-term).

7. Grades

Written and Coding HW: 60% = 6×10%.

Discussion coding work: 7% = 7×1%.

Exams: better 20%, worse 13%.

Tentative Final Grades: A (>90), B(80-89), C(70-79), NR (<70).

Remark. There is no need to ask me to round up your grade as I will consider this option for everyone. Rounding up to a higher letter grade is only possible when the following conditions are simultaneously met: 1) your current numerical grade is at most 1.00 point away (up to two digits past the dot) from a letter grade change, i.e., 89.00-89.99, but not 88.99; 2) the two exam scores do not differ by more than 20 points.

8. Special Arrangements

If you need course adaptations or accommodations because of a disability, or if you have medical information to share with me, please make an appointment with me as soon as possible. My office location and office hours are listed above. If you have not already done so, students who believe that they may need accommodations in this class are encouraged to contact the Office of Accessibility Services (OAS) as soon as possible to ensure that these accommodations are implemented in a timely fashion. The OAS is in Unity Hall, (508) 831-4908. Students who need accommodations for exams are required to make the arrangements to take these exams at the Exam Proctoring Center (EPC) on the day of the exam.

9. Additional Policies

9.1. **Electronic Policy.** All lectures (audio and video) are captured through course capture and can be found on the course Canvas page. **NO** recording of audio or video by students is allowed during lecture or during discussion. Laptops, phones, and tablets should be turned off during the lecture and conference sessions. If you take notes (typing/stylus only) using these devices during lecture, then you should sit somewhere in the room where your screen activity is not distracting to your neighbors.

9.2. **Exam Policy.** Prior to the start of each exam, you must place all of your belongings (e.g., cell phone, study materials, smart watches, etc.) in your backpack and under your desk, so that no items are visible during the exam. **All quizzes this term are closed book, closed notes, and no electronic devices are permitted.** Cheating is an extreme offense. Students involved in plagiarism will be reported to the Dean without grace periods and negotiations.

9.3. **AI Policy .** AI engines such as ChatGPT have been prevalent in our daily life. One unfortunate use is in providing solutions to Homework Problems with (near) impunity due to the random nature of such engines. Artificial intelligence (AI) language models, such as ChatGPT, and online assignment help tools, such as Chegg®, are examples of online learning support platforms: they can not be used for course assignments and will never be permitted in this course. Please review the WPI webpage "What is Academic Dishonesty". **If the Instructor even develops the doubts that a student has plagiarized assignment solutions from AI-engine generated texts, he will fail the students without tolerance.** The proper way of using ChatGPT is to attach the link associated with the generated text at the end of the assignment, so the grader is one click away from it.

The Instructor and the TA reserve the rights to withhold the alleged assignment(s) and conduct an oral exam on the material therein to verify that the student is capable of matching the understanding displayed in the solutions.