MA 3257/CS 4032 - Numerical Methods for Linear and Nonlinear Systems

C-Term 2025

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Class Times and Locations

Lecture: Kaven Hall 116, MTRF: 1:00-1:50PM

Discussion: Kaven Hall 116, W: 1:00-1:50PM

Office Hours: **No appointment needed**

Tilley (SH 419): R: 2:00-3:50pm, F: 10:00-10:50am

Balin: (MTC): T 1:00-2:00pm, (Math Lounge): T: 2:00-3:00pm , W: 12:00 -1:00 pm

Course Description: This course provides an introduction to modern computational methods for linear and nonlinear equations and systems and their applications. Topics covered include solution of nonlinear scalar equations, direct and iterative algorithms for the solution of systems of linear equations, solution of nonlinear systems, and the eigenvalue problem for matrices. Error analysis will be emphasized throughout. Recommended background: MA 2071. An ability to write computer programs in a scientific language is assumed.

Prerequisites: Recommended background: MA 2071. An ability to write computer programs in a scientific language is assumed. In our class, we will be using MATLAB for numerical computations and coding labs.

MATLAB Onramp: A web-based interactive tutorial on MATLAB and the desktop environment.

Installation instructions for Matlab at WPI

System Requirements for Matlab

Recommended Texts:

Numerical Linear Algebra: A Concise Introduction with MATLAB and Julia, F. Bornemann (Translated by W. Simson) (ISBN: 978-3-319-74221-2), (2018).

Numerical Analysis, Tenth Edition, R.L. Burden, D.J. Faires, A.M. Burden (ISBN-13: 978-1305253667) (2015)

Classroom Culture

Behavior Expectations: As an upper-level mathematics course, collaborative learning and active engagement are expected. ***Collaborative learning*** meas that students collaborate together to learn the material in the course. ***Active engagement by students*** means that students accept the responsibility for their own learning of the material and do not perceive the instructor (professor/pla) as a source of all knowledge.

In order to meet these expectations, the classroom environment must be professional and supportive. Students are expected to treat each other with mutual respect, provide constructive feedback to other students, and to realize that as humans we all need guidance at times. **pectations: As an upper-level mathematics course, collaborative learning es.**

Course Content Organization

- The course content is organized in the following *themes*

1. *Linear Systems:* Direct Methods, Factoring, Conditioning, Iterative Methods
2. *Nonlinear Systems:* Root-finding, Optimization

- To learn the material fully, we apply the following *methods*

1. *Theory*: Extending concepts from analysis and linear algebra to algorithms intended to approximate their computational analogues. Requires mathematical proof and rigor.
2. *Practice:* Developing scripts in MATLAB to implement these algorithms and verify that the results are consistent with the mathematical theory.

Class Meeting Format

- *Lectures (MTRF):* Initial presentation of the material which will vary in terms of theory and practice. Students are expected to attend lecture to participate in the class, such as asking questions. Note that attendance will be taken during the first week of class in order for the instructor to learn all of your names, but attendance is not part of the final grade (see below). All lectures are recorded on Echo 360, my lecture notes are typically posted on Canvas a day before lecture to help students prepare, and the lecture slides used in the lecture will be posted on Canvas after the lecture. **Students are not expected to be expert in the topic of the lecture by the end of the lecture.**
- *Discussion (W):* Opportunity for students to ask specific questions on a homework, laboratory, or any relevant topic covered in the class so far, and a space for students to work on their assignments with the PLA present. No lectures are given during this time, and no common group activity is scheduled or planned.

Course Grades and Policies

Grades will be determined based on homework assignments, coding labs and two in-class exams

- ***Homework: 15%:*** These assignments consist of questions which are either 1. Good examples of a question which can appear on a future exam, or 2. Questions that require students to synthesize two or more different concepts to arrive at the answer. Written homework will be turned in via an upload to **Gradescope** for each assignment. Each assignment is worth 100 points. Due dates for assignments can be found on the Canvas course site. **The lowest two homework scores are not counted toward the final grade.**

  Students must use a document scanner (e.g. Microsoft Lens, Camscanner, etc.) that produces a high-quality black/white or grayscale scan into a PDF file. All written assignments should be neatly written, and the quality of the scan must make the work legible (HINT: red ink scans really well). . **Assignments that cannot be read will result in the student submitting another scan after the due date, and receiving a 5 point deduction in the assignment grade.**

- ***Labs: 15%***: These asynchronous assignments will be a combination of programming assignments and written interpretation of the results of the programming exercises. The programming exercises will be through the Matlab Grader portal, while a series of summary questions will be posted on Canvas. These questions aim to find how well they understand the concepts and implications of the lab assignment. Both the Matlab Grader and Canvas portions will have the same due date. **All laboratory assignments count toward the final grade.**

- ***Two Exams: 70%*** Two in-class exams will take place at the dates below. The topics are the specific themes described above. **The contribution to the final grade depends on which exam a student got a better score:**

  - Higher score exam: 37%
  - Lower score exam: 33%

  The exam dates are:

  - Exam 1: February 7, 2025
  - Exam 2: March 7, 2025

- ***Grading Policy*** Students have **two business days** to contact the instructor about potential errors in grading any assignment (homework, labs, exams) after receiving their graded work. Beyond this time, the grade on that assignment is final.

- ***Make-Up Assignment Policy:*** The acceptance of late work for credit will be determined on a case-by-case basis. Students who feel they need 'ditional time to complete a homework or lab assignment need to let the instructor know **at least 25 hours prior to the due date/time** in order to get the terms under which the student can receive full credit.

- ***Accommodations:*** Students with approved academic accommodations should plan to submit their accommodation letters through the Office of Accessibility Services Student Portal. Should you have any questions about how accommodations can be implemented in this particular course, please contact me as soon as possible. Students who are not currently registered with the Office of Accessibility Services (OAS) but who would like to find out more information about requesting accommodations, documentation guidelines, and what the accommodated interactive process entails should plan to contact OAS either by email: AccessibilityServices@wpi.edu, by phone (508) 831-4908, or by stopping by the office on the 5th floor of Unity Hall.

Academic Integrity and WPI Policies

All students are expected to be familiar with and adhere to WPI's policy on academic integrity (i.e., no cheating, fabrication, facilitation, or plagiarism). Please refer to the WPI Academic Honesty Policy within the Student Code of Conduct (https://www.wpi.edu/about/policies/academic-integrity). Academic integrity violations will be prosecuted according to the university's policy. For more details as to what constitutes academic dishonesty, please see https://www.wpi.edu/about/policies/academic-integrity/dishonesty.

Students are responsible to complying with all of WPI's policies at https://www.wpi.edu/about/policies

Lecture Schedule

Click on image to enlarge schedule in separate tab.

Sun	Monday	Tuesday	Wednesday	Thursday	Friday
1/12			Introduction,, Error, Big-O Notation	Floating Point Representation, Computer Arithmetic	Calculus Review, Convergence of Sequences, Tolerance
1/19	Martin Luther King Day: No class	Recurrence Relations: Stability, Fixed Point Iteration, Contraction		Rates of Convergence, Bisection,	Elements of Programming for Scientific Computing, Newton's Method
1/26	Secant Method,, Method of False Position	Finding Minima without Derivatives; Golden Section Search		Finding Minima without Derivatives: Successive Quadratic Interpolation	Introduction: Discrete Least Squares, Linear Algebra Review: Ax=b
2/2	Gaussian Elimination	Gaussian Elimination - Pivoting		Review for Midterm (1/15-1/31 Topics)	Midterm Exam
2/9	Special Types of Matrices	Direct Factorization		Iterative Methods Intro	Wellness Day: No class
2/16	Snow Day - No class	Ill-Conditioning and Condition number		Iterative Methods - Jacobi & Gauss-Seidel	Iterative Methods - SOR
2/23	Eigenvalue Problems	Eigenvalue Problems - Power Method		Reading Day: No class	Nonlinear Systems- Fixed-Point Methods
3/2	Nonlinear Systems - Newton-Raphson Method	Nonlinear Systems - Steepest Descent and Learning Rate		Review for Final Exam (2/2-2/28 Topics)	Final Exam