MA 4473: Partial Differential Equations

B-Term, Fall 2024

Department of Mathematical Sciences

Worcester Polytechnic Institute

Instructor: Prof. B.S. Tilley, Department of Mathematical Sciences,
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(Not read after 7:00 pm; M-F: reply within 24 hrs; Sat/Sun: 48 hrs)

Office Hours (no appointment needed): R: 3:00-4:20pm
F: 1:30-2:50pm

Location: SH 207

Lecture Times: MTRF - 10:00 - 10:50 AM

Textbook: ***Introduction to Partial Differential Equations*****, P.J. Olver, Undergraduate Texts in Mathematics, Springer (2020)**
ISBN 978-3-319-02098-3

Note: PDF available to WPI students through the WPI Library Website

Additional References:

- ***Partial Differential Equations, An Introduction, Second Edition*****, W.A. Strauss, Wiley (2008)**
- ***Introduction to Partial Differential Equations with Applications*****, E.C. Zachmanoglou and D.W. Thoe, Dover (1987)**
- ***Elementary Applied Partial Differential Equations*****, R. Haberman, Pearson (2021)**
- ***Green's Functions and Boundary Value Problems*****, I. Stakgold, Wiley (2011).**

Course Description: The first part of the course will cover the following topics: classification of partial differential equations, solving single first order equations by the method of characteristics, solutions of Laplace's and Poisson's equations including the construction of Green's function, solutions of the heat equation including the construction of the fundamental solution, maximum principles for elliptic and parabolic equations. For the second part of the course, the instructor may choose to expand on any one of the above topics. This course will be offered in 2022-23, and in alternating years thereafter.

Provisional Outline of Topics (updated weekly)

Week 1: Review of Vector Calculus, First-Order PDEs, Quasi-Linear First-Order PDE's, Conservation Laws, Shocks and Weak Solutions
Week 2: Systems of First-Order PDEs, Wave Equation, d'Alembert's Solution
Week 3: Classification of Second-Order PDEs, Heat Equation: Derivation, Similarity Solution, Maximum Principle
Week 4: Theory of Distributions, Green's functions on the real line, Green's Identities, Laplace's Equation, Well-Posedness
Week 5: Free-space Green's function for Laplace, Maximum Principle, Representation Theorem, Method of Images, Poisson Kernel
Week 6: Higher dimensions: Domain of Influence, Method of Spherical Means
Week 7: Dispersion and Nonlinear Waves (Burgers' Equation, KdV)

Class Expectations: As an upper-level mathematics course, collaborative learning and active engagement are expected. *Collaborative learning* meas that students collaborate together to learn the material in the course. *Active engagement* by students means that students accept the responsibility for their own learning of the material and do not perceive the instructor (professor) as a source of all knowledge.

In order to meet these expectations, the classroom environment must be professional and supportive. Students are expected to treat each other with mutual respect, provide constructive feedback to other students, and to realize that as humans we all need guidance at times.

While attendance is not taken and does not count for the grade, students are responsible for all material covered in lecture and need to meet deadlines for homework assignments. Flexible arrangements will be considered on a case-by-case basis.

Students should also be able to communicate the mathematics they are learning, in both written and oral form, to others.

Grading:

- ***Homework*: (30% of final grade)** Each week, 1-2 homework assignments will be given that focus on the material covered that week. These problems will be due about 1 week after the lecture. Students are encouraged to help each other learn concepts, but are **forbidden** from copying proofs or solutions from other students or other sources. The average of the top 5 homework assignments determine this portion of the course grade.
- ***Midterm Exam:* (35 % of final grade) November 14, 2024** Topics for this exam will be the material covered in Weeks 1-3. The exam will be open book, open notes, but no internet access.
- ***Final Exam* : (35% of final grade) December 13, 2024** Topics of this exam will be the material covered in Weeks 4-7. The exam will be open book, open notes, but no internet access.
- **Generative AI:** Problems which are solved using generative AI (e.g. ChatGBT, Gemini, ..) will be marked with zero credit.

Student Accommodations

Students with approved academic accommodations should plan to submit their accommodation letters through the Office of Accessibility Services Student Portal. Should you have any questions about how accommodations can be implemented in this particular course, please contact me as soon as possible. Students who are not currently registered with the Office of Accessibility Services (OAS) but who would like to find out more information about requesting accommodations, documentation guidelines, and what the accommodated interactive process entails should plan to contact OAS either by email: AccessibilityServices@wpi.edu, by phone (508) 831-4908, or by stopping by the office on the 5th floor of Unity Hall.