Worcester Polytechnic Institute
Department of Mathematical Sciences
May 11, 2026 NAME: ____________________

General Comprehensive Exam
LINEAR ALGEBRA

Problem 1 Assume A is an $n \times n$ matrix diagonalizable over the complex numbers. Determine which of the following matrices are also diagonalizable. Explain briefly.

(i) $7A - \lambda I_n$ where λ is an eigenvalue of A

(ii) A^5

(iii) A^{-1}, assuming A is nonsingular

(iv) $\sin(A) = A - \frac{1}{3!}A^3 + \frac{1}{5!}A^5 - \frac{1}{7!}A^7 + \cdots$

Problem 2 Let V be a complex inner product space and $L : V \to V$ a linear transformation.

(i) Prove: $L = 0$ *if and only if* $\langle L(v), v\rangle = 0$ *for all* $v \in V$. *[HINT: Consider combinations such as* $u + v$ *and* $u - iv$*.]*

(ii) Is this statement still true if V is an inner product space over $\mathbb{R}$?

Problem 3 Let A and B be $n \times n$ matrices over a field $\mathbb{F}$. Use the Dimension Formula for the range and kernel of a matrix to prove that $AB = I$ implies $BA = I$. *[Do not use determinants.]*

Problem 4 Prove that every $n \times n$ matrix A with real entries is expressible as the sum of two $n \times n$ matrices both diagonalizable over the real numbers. *[HINT: Consider the eigenvalues of upper and lower triangular matrices.]*