Exercise 1:
Use the Fubini theorem to prove that
\[ \int_{\mathbb{R}^n} e^{-|\mathbf{x}|^2} d\mathbf{x} = \pi^{n/2} \]
Here \( \mathbf{x} = (x_1, x_2, \ldots, x_n) \). Hint: For \( n = 2 \), use polar coordinates.

Exercise 2:
Let \((X, \mathcal{A}, \mu)\) be a measure space, and \( f \) be in \( L^1(X) \). Let for all positive integers \( n \) set \( B_n = \{ x \in X : n - 1 \leq |f(x)| < n \} \).

(i). Show that \( \mu(B_n) < \infty \) for all \( n \geq 2 \).

(ii). Show that \( \sum_{n=2}^{\infty} n\mu(B_n) < \infty \).

(iii). Define \( C_n = \{ x \in X : n - 1 \leq |f(x)| \leq n \} \). Is the sum \( \sum_{n=2}^{\infty} n\mu(C_n) \) finite?

(iv). Show that \( \sum_{n=2}^{\infty} \sum_{m=2}^{n} \frac{m^2}{n^2} \mu(B_m) < \infty \).

(v). Show that for \( n \geq 2 \)
\[ \int |f|^2 1_{\{|f|<n\}} = \int |f|^2 1_{\{|f|<1\}} + \sum_{m=2}^{n} \int |f|^2 1_{B_m} \]
and infer that \( \sum_{n=1}^{\infty} \frac{1}{n^2} \int |f|^2 1_{\{|f|<n\}} < \infty \).

Exercise 3:
Prove or Disprove: Suppose that \( f, g : \mathbb{R} \to \mathbb{R} \), with \( f \) being a measurable function, and \( g \) being a continuous function. Then \( f \circ g \) is measurable. By definition, \((f \circ g)(x) := f(g(x))\),
that is, it is the composition of the two functions.