U-Substitution Integral Puzzles

Course: Calculus

Goals: To enable students to:
- Develop an understanding of the relationship between derivatives requiring the Chain Rule and their corresponding integrals requiring U-Substitution (prior to learning the U-Substitution process).

Prerequisite Skills:
- Differentiation using the Chain Rule,
- Basic antidifferentiation techniques/rules

Materials Required:
For each pair/small group of students, you will need:
- One set of puzzles 1 – 6, each photocopied on a different colored paper
- One Answer sheet
- Scissors

An Approach:
- Begin the class with a few derivative problems that require the use of the Chain Rule,
- Split the students into pairs or small groups,
- Have each group complete each puzzle and fill in the answer sheet,
- Once all puzzles are completed, have students answer the two questions at the bottom of the answer sheet to reflect on their process and the relationship between these integrands and their antiderivatives,
- Begin the process of integration using U-Substitution directly following this activity.

Alternate/Further Ideas:
- This can be done as a "station activity" with pre-cut puzzles for a shorter activity.
- Students can create puzzles of their own by working backwards from their own chain rule problems and

exchange these with other groups to complete (provide blank template if desired).

U-Substitution Puzzle Answer Sheet

<u>Directions:</u>
1. For one puzzle, cut out all puzzle pieces;
2. Arrange the pieces so that they form a correct equation (all pieces must be used; no pieces may be created/added);
3. Write the puzzle solution (i.e. the correct equation) in the answer space below;
4. Repeat process for remaining puzzles;
5. Answer the questions about the process your group used.

Puzzle	Solution (Equation)

1. Briefly explain your approach to solving the puzzles. Use complete sentences and correct mathematical vocabulary; include how you checked your solutions.

2. What appears to be the relationship between the integrand and the antiderivative for each puzzle solution? Be as specific as possible.

Puzzle 1

$\int$	dx	$\dfrac{1}{4}$
$x^2 - 10$	$x^2 - 10$	2
x	$+$	$=$
$($	$($	$)^3$

$)^4$	c	

Puzzle 2

$\displaystyle\int$	$d\theta$	5
$\sin^5\theta$	$\sin^4\theta$	$\cos\theta$
c	$+$	$=$

Puzzle 3

$\displaystyle\int$	dx	$\dfrac{1}{2}$
e^{x^2}	e^{x^2}	c
x	$+$	$=$

Puzzle 4

$\displaystyle\int$	dx	15
$x^3 + 5$	$x^3 + 5$	x^2
c	$+$	$=$
$($	$($	$)^4$

$)^5$		

Puzzle 5

$\int$	dx	x^3
$1 + x^4$	$1 + x^4$	4
$-$	$+$	$=$
$($	___	___

$)^2$	c	1

Puzzle 6

$\displaystyle\int$	dx	$\dfrac{1}{3}$
$\ln x$	$\ln x$	x
c	$+$	$=$
$($	$($	$)^2$

$)^3$	——	