WORCESTER POLYTECHNIC INSTITUTE

Tenth annual invitational math meet october 23, 1997 individual exam question sheet

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

1	Determine	the	number	of	distinct	prime	factors	of	39039
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If
$$x^a x^b = 1$$
 and $x \neq \pm 1$, evaluate $4a - b^2 + a^2 + 4b - 10$.

In a geometric sequence of real numbers, the sum of the first two terms is 7 while the sum of the first six terms is 147. What is the sum of the first four terms?

The 1967 World Almanac indicated that Rhode Island had a population of 892,709 with a population density (population/sq. mile) of 843.8, while Worcester County in Massachusetts had a population of 583,228 with a population density of 384.7. Which is larger in area, Worcester County or Rhode Island?

Mina received \$200 to spend in Paris. The exchange rate is 5.88 French francs to the dollar. There is a 4.5% tax included in the prices in France. She knows, however, that being a U.S. citizen, she will not have to pay the tax. What is the highest price (in French francs) of an article that Mina can buy?

Each student in a college must go for lottery to obtain housing. One out of every five applicants (selected at random) is offered housing on campus. If a student enters the lottery for two consecutive years, what are her chances of getting an offer for housing?

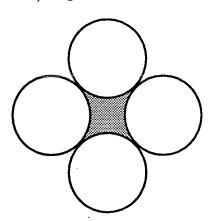
What is the 1997th digit in the decimal representation of the number 1/13?

8 A polynomial has remainder 8 when divided by x-2, remainder 3 when divided by x-3, and remainder -6 when divided by x-4. Find the remainder when the same polynomial is divided by (x-2)(x-3)(x-4).

9 A function f, defined for all non-zero real numbers x, satisfies 3f(x) + 4f(1/x) = 5x. Find all x for which f(x) - f(-x) = 0.

If an arc of 45° on Circle I has three times the length of an arc of of θ° on Circle II, and the area of Circle I is four times the area of Circle II, find θ (in degrees).

Four circles of the same size are arranged as shown in the adjoining figure. If the total area of the four circles is 36π , what is the diameter of the largest circle that can be drawn in the enclosed (shaded) region?



NAME	
SCHOOL	DOLUTIONS!

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QUESTION		ANSWER	SCORE
1		4	
2	-	-10	
3		35	
		ESTER	
4	Coun	ιτγ	
	# co	DRRECT × 1	=

QUI	ESTION	ANSWER	SCORE
5	12	28.92	
6	Q,	/25	
7		2	
8	$-2x^2$	+5×+6	
	# CC	ORRECT × 2	=

QUESTION		ANSWER	SC	CORE	
9	<u>+</u>	2/√3			
10		30			
11	6.	T2 -6			
	#	CORRECT X	3 =		

Individual	Total