DIRECTIONS: Please write your answers on the Team Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators MAY NOT be used.

1 The points A,B,C and D are collinear where AB, BC and CD are the diameters of three circles, each of radius 10. Let the line AE be tangent to the circle through C and D at the point E, as shown in the diagram. The line AE intersects the circle through B and C at the points F and G as shown. Find the length of the chord FG.
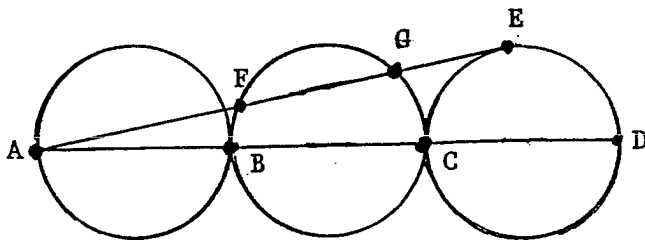
2 Given that $\dfrac{a}{b+c} + \dfrac{b}{a+c} + \dfrac{c}{a+b} = 2$, simplify the expression $\dfrac{a^2}{b+c} + \dfrac{b^2}{a+c} + \dfrac{c^2}{a+b}$.

3 Let $f(x) = 3 + |x-4| + 2|x-6|$, $2 \le x \le 8$. Find the area of the planar region determined by the graphs of $y = f(x)$, $y = 1$, $x = 2$ and $x = 8$.

4. Consider hexagonal array shown at the right. The nth <u>hex number</u>, h_n, is defined as the total number of dots in the first n layers. The first seven hex numbers are 1, 7, 19, 37, 61, 91, 127. Find h_{50}, the 50th hex number.
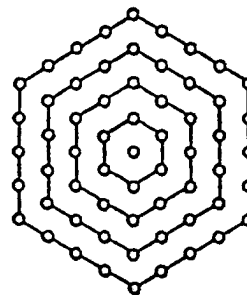
5. Let $g(x) = \sqrt{x + 2\sqrt{x-1}} - \sqrt{x - 2\sqrt{x-1}}$, $x \geq 1$. Compute $\sum_{k=1}^{1000} g(k)$.

6. An ant moves across the top of a square at 4 mph, down the right side at 3 mph, across the bottom at 2 mph, and up the left side at 1 mph. What is the average speed of the ant, in mph, for one complete trip around the square?

2

7 How many odd numbers between 1000 and 9999 have distinct digits?

8 Suppose that at the end of any year, a unit of money has lost 20% of the value it had at the beginning of that year. Find the least integer n such that at the end of n years the unit of money will have lost at least 80% of its initial value. (Assume $\log_{10} 2 = 0.301$.)

9 Twin primes are prime numbers which differ by 2 (17 and 19, or 29 and 31, for example). There are many twin primes, but there is only one triplet of primes: 3, 5 and 7. Which of the following best express the reason(s) why there cannot be any other triplet of primes?

  a One of the three integers would be a perfect square (in the sequence 23, 25, 27, for example, 25 is a perfect square),

  b In any sequence of three integers, if successive integers differ by 2, then one of the three integers is divisible by 3.

  c If one divides the first integer in such a sequence by the last, the result is less than 1.

  d $3^2 + 5^2 < 7^2$ while this is not true for large integers. For example $(23)^2 + (25)^2 > (27)^2$.

3

$\boxed{10}$ What is the coefficient of a^4b^2c in the expansion of $(a + 2b + c)^7$?

$\boxed{11}$ Find the number of lines in a three-dimensional rectangular coordinate system that pass through five distinct points of the form (i, j, k), where i, j, k are positive integers not exceeding 5.

$\boxed{12}$ In the sequence of numbers 1, 4, 3 ... each term after the first two is equal to the term preceding it minus the term preceding that. Find the sum of the first one hundred terms of the sequence.

4

13 Compute the value of $\left(1 - \dfrac{3}{7}\right)\left(1 - \dfrac{3}{8}\right)\left(1 - \dfrac{3}{9}\right)\left(1 - \dfrac{3}{10}\right)\cdots\left(1 - \dfrac{3}{20}\right).$

14 Let z be a complex number of magnitude 1, $z \neq -1$. Let $i = \sqrt{-1}$, and let c and d be real numbers with $c + di = \dfrac{1}{1 + z}$. Find c.

5

SCHOOL _____ *SOLUTIONS* _____

WORCESTER POLYTECHNIC INSTITUTE

Eleventh ANNUAL INVITATIONAL MATH MEET
OCTOBER 22, 1998
TEAM EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE
1	16	
2	$a+b+c$	
3	42	
4	7,351	
5	1,998	
6	$\frac{48}{25}$ or 1.92	
7	2,240	

QUESTION	ANSWER	SCORE
8	8	
9	6	
10	420	
11	109	
12	7	
13	$\frac{1}{57}$	
14	$\frac{1}{2}$	

# CORRECT × 3 =	
Individual Totals	

Team Total