DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

1. For what real values of x does the following identity hold?

$$\sqrt{x + 2\sqrt{x-1}} + \sqrt{x - 2\sqrt{x-1}} = 2\sqrt{x-1}$$

2. Solve for t so that

$$\frac{9^{n+2} + 81^{2n}27^{1-2n}}{\sqrt[3]{8^2 3^{6n}}} = 3^t$$

3. Alice says she has twice as many brothers as sisters, but her brother has twice as many sisters as he has brothers. How many sisters does Alice have?

4. Find the smallest positive integer in the set

$$\{96x - 270y \mid x, y \text{ are integers}\}$$

$\boxed{5}$ If $\tan(\alpha) + \cot(\alpha) = m$, express $\tan^4(\alpha) + \cot^4(\alpha)$ in terms of m.

$\boxed{6}$ Consider the parabola $y = \dfrac{1}{8}x^2$. A ray comes down vertically, parallel to the y axis, and reflects off the parabola at the point whose x coordinate is 3. What is the x coordinate of the point where it strikes the parabola a second time?

$\boxed{7}$ In the following diagram, which is **not** drawn to scale, the shaded area is 32π. What is $\sin(x)$?
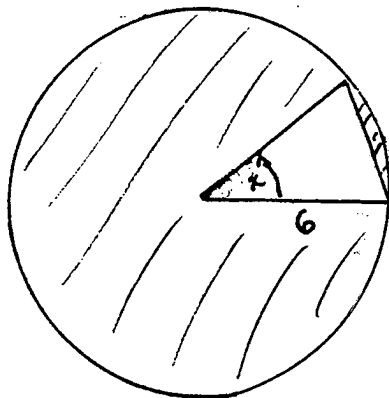
$\boxed{8}$ Consider a triangle in the coordinate plane with vertices $(1,1), (2,3)$ and $(x,2)$ where $x > 1$. What is the area of the triangle?

9. Let $a = \underbrace{1111\ldots1111}_{1998\,\text{digits}}$ and $b = \underbrace{2222\ldots2222}_{999\;\text{digits}}$. Compute $\sqrt{a - b}$

10. If a baseball team has a probability P of beating its opponent in any one game, independently of what happens in any other games, what is the probability it wins a 5 game series? (a winner is the first team to win 3 games)

11. Consider a circle with n points on its boundary. How many possible points of intersection are there between the chords inside the circle? Assume that no three chords intersect at the same point.

NAME _Answers- Individual_

SCHOOL _____

WORCESTER POLYTECHNIC INSTITUTE

TWELFTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 21, 1999
INDIVUAL EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE	QUESTION	ANSWER	SCORE
1	$x > 1$		5	$m^4 - 4m^2 + 2$	
2	$t = 3$		6	$-\frac{16}{3}$	
3	1		7	$\frac{2\pi}{9}$	
4	6		8	$\left| x - \frac{3}{2} \right|$ or $\left| \frac{3}{2} - x \right|$	
	# CORRECT $\times$ 1 =			# CORRECT $\times$ 2 =	

Individual Total

QUESTION	ANSWER	SCORE
9	$\underbrace{333 \cdots 333}_{999 \text{ digits}}$	
10	$6P^5 - 15P^4 + 10P^3$	
11	$\binom{n}{4} = \frac{n(n-1)(n-2)(n-3)}{4!}$	
	# CORRECT $\times$ 3 =	