WORCESTER POLYTECHNIC INSTITUTE

TWELFTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 21, 1999
TEAM EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Team Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators **MAY NOT** be used.

$\boxed{1}$ Let S be the set of all ordered pairs of integers. On S define a multiplication, $\odot$, such that

$$(a, b) \odot (c, d) = (ac + 2bd, ad + bc)$$

where the operations on the right are usual multiplication and addition of integers. What is the identity element for the operation $\odot$ on S?

$\boxed{2}$ What is the area of a segment of a circle of radius 2 bounded by a 15 degree arc and its chord?

$\boxed{3}$ What is the 9th power of the matrix

$$\begin{pmatrix} \dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \\ -\dfrac{1}{2} & \dfrac{\sqrt{3}}{2} \end{pmatrix}$$

1

4 The roots of $px^2 + qx + 1$ are $1/3$ and $1/2$. What are the roots of $qx^2 - px - 1 = 0$?

5 Express $\displaystyle\sum_{k=-3}^{k=+3} \frac{1}{9^k}$ as a base 3 number.

6 For a point (x, y) on the graph of

$$4x^2 + 9y^2 - 16x + 18y - 11 = 0$$

what is the sum of the distances from (x, y) to the points

$$(2 + \sqrt{5}, -1) \text{ and } (2 - \sqrt{5}, -1)?$$

[7] At a recent Olympics, it was found that the average age of all males was 19 while the average age of all females was 15. If the average age of all participants was 17 years, 6 months, what was the ratio of females to males?

[8] Name the 76th natural number not divisible by 2 or 7.

[9] Solve for $x > 0$.

$$2\log_4 x = \log_4 4 + \log_4(4x + 9)$$

10 Solve for x between 0 and π.

$$4\sin^4(x) = 4\cos^2(x) - 1$$

11 Find the point in the region defined by
$x, y \geq 0$,
$y \leq 15$,
$x \leq 15$,
$2x + 3y \leq 57$,
where the function $f(x, y) = 6x + y$ is greatest.

12 Solve the equation $x^3 - [x] = 3$ where $[x]$ is the greatest integer less than or equal to x.

4

$\boxed{13}$ Two chess players play a series of matches. The "winner" is the first to win two in a row. If player A has probability P of beating player B in any one match, independently of what happens in other matches, what is the probability A wins?

$\boxed{14}$ Four of the eight vertices of a cube are vertices of a regular tetrahedron. Find the ratio of the surface area of the cube to the surface area of the tetrahedron.

SCHOOL Answers – TEAM

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TEAM EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE	QUESTION	ANSWER	SCORE
1	$(1, 0)$		8	177	
2	$\frac{\pi}{6} - \sqrt{2 - \sqrt{3}}$		9	18	
3	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$		10	$\frac{\pi}{4}$	
4	$-1, -\frac{1}{5}$ (either order)		11	$(15, 9)$	
5	1010101.010101		12	$4^{1/3}$ or $\sqrt[3]{4}$	
6	6		13	$(2-p)\sum\limits_{k=2}^{\infty} p^k (1-p)^{k-2}$	
7	$3:5$		14	$\sqrt{3}$	

Team Total

# CORRECT × 3 =	
Individual Totals	