WORCESTER POLYTECHNIC INSTITUTE

THIRTEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 26, 2000
INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

[1] A ball is dropped from 300 feet and bounces off level ground. The ball rebounds to 2/3 of its previous height after each bounce. Find the total distance traveled by the ball when it hits the ground the 3rd time.

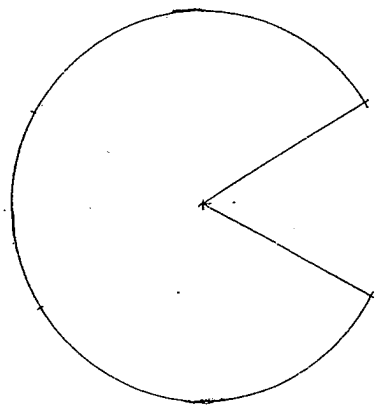
[2] What is the slope of the line perpendicular to the line satisfying the following equation?

$$\frac{x-2}{-y+3} = -\frac{7}{8}$$

[3] It is known that the perimeter of a semi circular region, measured in centimeters, is numerically equal to its area, measured in cm^2. What is the radius of the semicircle?

[4] Given: $f(0) = 3$; $f(n+1) = 2f(n) + 3$, what is $f(3)$?

[5] At a given moment, Pac-Man's mouth is open in such a way that the angle formed between his lips is 60 degrees. If one of his lips is 2 cm long, what is his total perimeter in cm? Assume he is circular.



6 What is the probability that at least two students in a class of size n, where $0 < n <= 366$ have the same birthday? Assume 365 days in a year.

7 What is

$$\sum_{j=0}^{30} \sum_{k=1}^{100} \left(\frac{1}{k} - \frac{1}{k+1} \right)?$$

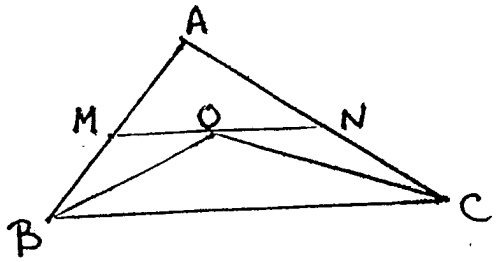
8 If $\log_9(\log_3(\log_2(x))) = 0$, then $x^{-1/2}$ is equal to what?

9 How many different letter arrangements can be made from the word:

COMPETITION ?

10 In triangle ABC, point F divides side AC in the ratio 1:2. Let E be the point of intersection of side BC and AG where G is the midpoint of BF. Point E now divides side BC in what ratio? (Please express as $B : C$.)

11 In the following picture, BO bisects $\measuredangle$CBA, CO bisects $\measuredangle$ACB, and MN is parallel to BC. If AB = 12, BC = 24, and AC = 18, what is the perimeter of triangle AMN?



NAME Solutions

SCHOOL Individual

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INDIVIDUAL EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE
1	$966^{2/3}$ or $\frac{2900}{3}$	
2	$-\frac{7}{8}$	
3	$\frac{2(\pi+2)}{\pi}$ or $2+\frac{4}{\pi}$	
4	21	
	# CORRECT × 1 =	

QUESTION	ANSWER	SCORE
5	$\frac{10\pi}{3}+4$ or $\frac{10\pi+12}{3}$	
6	$1-\frac{(365)(364)\cdots(365-n+1)}{(365)^n}$	
7	$31\left(1-\frac{1}{101}\right)$ or $\frac{3100}{101}$	
8	$\frac{1}{\sqrt{8}}$ or $\frac{1}{2\sqrt{2}}$	
	# CORRECT × 2 =	

QUESTION	ANSWER	SCORE
9	$\frac{11!}{8}$ or 4,989,600	
10	1:3	
11	30	
	# CORRECT × 3 =	

Individual Total
