

WORCESTER POLYTECHNIC INSTITUTE

THIRTEENTH ANNUAL INVITATIONAL MATH MEET

OCTOBER 26, 2000

TEAM EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Team Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators **MAY NOT** be used.

1 How many x values satisfy the equation $x \cos(x) - \sin(x) = 0$ for $-8 \leq x \leq +8$?

2 Express the numbers $22_3 + 161_7$ in binary (base 2). The subscripts (3 and 7) represent the base of each number.

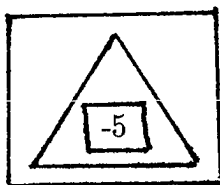
3 Let \boxed{n} be defined as:

$$\boxed{n} = \begin{cases} n & \text{if } n \leq 0 \\ 2n + 3 & \text{if } n > 0 \end{cases}$$

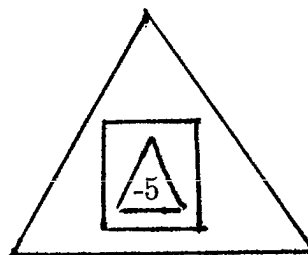
let $\triangle n$ be defined as

$$\triangle n = \begin{cases} |n - 1| & \text{if } n \leq -5 \\ |n^2| & \text{if } -5 < n < 5 \\ 3n - 4 & \text{if } n \geq 5 \end{cases}$$

What is

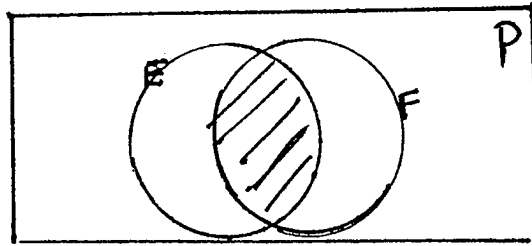


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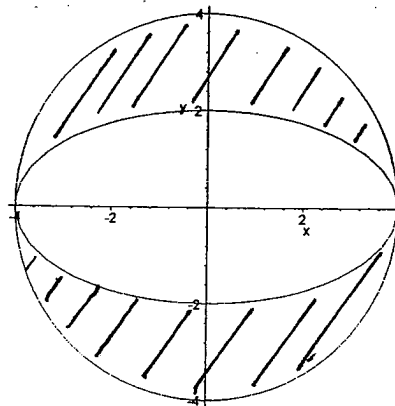
?

- 4] Which of the following expresses the shaded set shown? Choose ALL that apply.



(The notation \bar{A} means the complement of A relative to P)

- a) $E \cap F$ e) $\overline{\overline{F} \cup \overline{E}}$
 b) $E \cup F$ f) $\overline{\overline{F} \cap \overline{E}}$
 c) $P \cap (E \cap F)$ g) $(\overline{\overline{F} \cup \overline{E}}) \cup P$
 d) $P \cup (E \cap F)$ h) $(\overline{\overline{F} \cap \overline{E}}) \cap P$
- 5] What two whole numbers, neither containing any zeros, when multiplied together equal exactly 1,000,000,000?
- 6] Shown below is an ellipse inscribed in a circle. What is the area of the shaded region?



- 7] The sum of all but one of the interior angles of a convex polygon equals 700° . What is the value of the remaining angle?

8] In the sequence of 1, 3, 2, -1, ... each term after the first two is equal to the term preceding it minus the term preceding that. The **sum** of the first one hundred terms of the sequence is equal to what?

9] For what real numbers x is the quantity $\sqrt{x^3 + x^2 - 10x + 8}$ a real number?

10] In the triangle ABC , D is the midpoint of AB , E is the midpoint of DB ; and F is the midpoint of BC . If the area of triangle ABC is 96, what is the area of triangle AEF ?

11] Evaluate

$$\log_{10}(\tan 1^\circ) + \log_{10}(\tan 2^\circ) + \log_{10}(\tan 3^\circ) + \dots + \log_{10}(\tan 88^\circ) + \log_{10}(\tan 89^\circ)$$

12] If a and b are positive integers and

$$(2^{ab} - 1) = (X)(2^{ab-a} + 2^{ab-2a} + 2^{ab-3a} + \dots + 2^a + 1)$$

What is X ?

13] How many points with integer coordinates does the graph of

$$3x + 7y = 1$$

pass through?

14] A road which is level passes 12' above another level road at right angles to it. A bicyclist rides along the lower road at a speed of 9 feet per second. At the exact instant the bicycle passes underneath the upper road, a car passes overhead going four times as fast as the bicycle. One second later, how far apart are they?

SCHOOL Solutions- TEAM

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 TEAM EXAM ANSWER SHEET

QUESTION	ANSWER	SCORE	QUESTION	ANSWER	SCORE
1	5		8	5	
2	1100100 ₂		9	-4 ≤ x ≤ +1 and x ≥ 2 or [-4, 1] ∪ [2, ∞)	
3	15 - 41 or -26		10	36	
4	a, c, e		11	0	
5	512 and 1,953,125 2 ⁹ or 5 ⁹		12	2 ^a - 1	
6	8π		13	an infinite number or ∞	
7	20° or π/9		14	39'	

Team Total

# CORRECT × 3 =	
Individual Totals	