WORCESTER POLYTECHNIC INSTITUTE

FIFTEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 17, 2002
INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

[1] What is the distance from the point $(4, 12)$ to the line $y = -2x + 10$?

[2] A square is inscribed in an equilateral triangle as shown. Find the area of the shaded region.

[3] If $A = \begin{pmatrix} 2 & 1 \\ 6 & a \end{pmatrix}$, find a value for a so that A^2 is symmetric.

[4] What is the shaded area?

[5] For the function $f(x) = x^2 + 6x - 84$ find a "fixed point" of f; that is, a number x such that $f(x) = x$.

[6] Find a formula for the nth term in a sequence whose first four terms are

$$-\frac{1}{3}, \quad \frac{8}{9}, \quad -\frac{27}{19}, \quad \frac{64}{33}, \ldots$$

[7] The numbers of cubic feet in the volume of a cube is the same as the number of square inches of its surface area. What is the length of an edge of it expressed in feet?

[8] If f is a function defined by

$$f(t) = \frac{t}{1-t} \quad t \neq 1$$

what is a definition of the inverse function of f?

[9] An ellipse is described by $\frac{x^2}{100} + \frac{y^2}{75} = 1$. A ray crosses the x-axis at $(-5, 0)$, reflects off the ellipse at $\left(5, \frac{15}{2}\right)$ and then crosses the x-axis again. Where does it cross the x-axis the second time?

[10] What is the sum of the first twenty powers of 2?

[11] A piece of string is cut in two at a random point. What is the probability that the larger piece is at least x times as large as the shorter piece (where $x \geq 1$)?

NAME Key

SCHOOL

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QUESTION	ANSWER	SCORE
1	$\sqrt{20}$ or $2\sqrt{5}$	
2	$36\sqrt{3} - \frac{432}{7+4\sqrt{3}}$ or $\frac{252\sqrt{3}}{7+4\sqrt{3}}$	
3	$a = -2$	
4	$3\pi - \frac{9\sqrt{3}}{2}$	
	# CORRECT × 1 =	

QUESTION	ANSWER	SCORE
5	7 or -12	
6	$(-1)^n \frac{n^3}{1+2n^2}$	
7	864	
8	$f^{-1}(t) = \frac{t}{1+t}$ (or any other variable)	
	# CORRECT × 2 =	

QUESTION	ANSWER	SCORE
9	$x = 5$ or $(5, 0)$	
10	$2^{21} - 2$	
11	$\frac{2}{(x+1)}$	
	# CORRECT × 3 =	

Individual Total