WORCESTER POLYTECHNIC INSTITUTE

### FIFTEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 17, 2002
TEAM EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Team Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators MAY NOT be used.

1. Find the area of the annulus between the two concentric circles given that the length of any cord of the outer circle, tangent to the inner circle, is 40 cm.
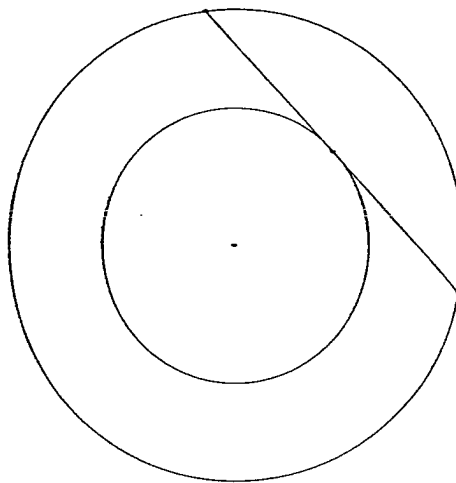
2. Gamma rays strike a pentagonal flat surface with uniform probability. There is a disk on the surface. What should its diameter be so that the probability of striking it is 1/3?
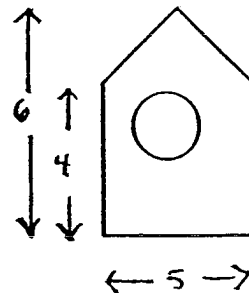

3. How many roots in $[0, 4]$ does the following function have?

$$f(t) = (t^3 - 7t^2 - 34t + 40)\sin^2\left(\frac{7t}{2}\right)$$

4. What is i^i in standard complex form? $\left(i = \sqrt{-1}\right)$

5. Assuming the radius of the Earth to be 1 unit, what are the Cartesian Coordinates of a location whose longitude is $30°w$ and whose latitude is $60°$ north? Assume that the equator is in the $x - y$ plane, the z axis goes through the north and south poles, and the $x - z$ plane passes through $0°$ longitude.

6. For the function $f(x) = x^2 + 1$, find $f(f(\frac{1}{z}))$.

7. Find a 3rd point so an equilateral triangle is formed where two of the points are $(2, 1)$ and $\left(5\sqrt{3} + 2, 6\right)$

$\boxed{8}$ Find all cube roots of -27.

$\boxed{9}$ In the following graph, what is the ratio of the area of triangle T_1 to the area of triangle T_2?
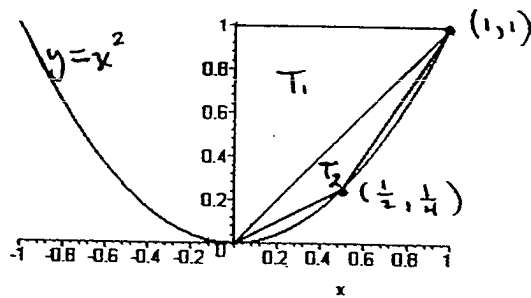
$\boxed{10}$ In the following drawing, what is the relationship between angle α and angle θ?
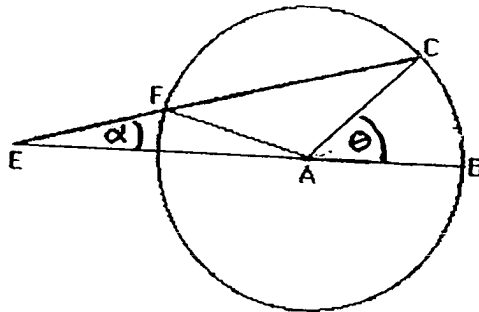
$\boxed{11}$ What is the sum of the following series?

$$41 + 45 + 49 + 53 + \ldots + 437.$$

$\boxed{12}$ If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ for $-1 < x < +1$ then $f\left(\frac{3x+x^3}{1+3x^2}\right)$ in terms of $f(x)$ is what?

$\boxed{13}$ The area of a rectangle remains unchanged when it is made 2 1/2 inches longer and 2/3 inch narrower, or when it is made 2 1/2 inches shorter and 4/3 inch wider. What is its area in square inches?

$\boxed{14}$ A triangle has sides with lengths of 25, 15 and 20. It is inscribed in a circle. What is the radius of the circle?

SCHOOL Key

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TEAM EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE	QUESTION	ANSWER	SCORE
1	$400\pi\, cm^2$		8	$-3,\ \frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$	
2	$2\sqrt{\frac{25}{3\pi}}$ or $\sqrt{\frac{100}{3\pi}}$		9	$4 : 1$	
3	6		10	$\alpha = \frac{\theta}{3}$	
4	$e^{-\pi/2}$		11	$23,900$	
5	$x = \frac{\sqrt{3}}{4},\ y = -\frac{1}{4},\ z = \frac{\sqrt{3}}{2}$ or $\left(\frac{\sqrt{3}}{4},\ -\frac{1}{4},\ \frac{\sqrt{3}}{2}\right)$		12	$3f(x)$	
6	$\frac{1}{z^4} + \frac{2}{z^2} + 2$		13	20 square inches	
7	$(2, 11)(5\sqrt{3} + 2, -4)$		14	$\frac{25}{2}$	

Team Total

# CORRECT $\times$ 3 =	
Individual Totals	