WORCESTER POLYTECHNIC INSTITUTE

SEVENTEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 20, 2004
TEAM EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Team Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators MAY NOT be used.

1. A circle of radius 1 is rolling without slipping around a circle of radius 3. How many full rotations will the small circle make while traveling once around the larger circle?

2. A circle is surrounded by circles of the same size and a big circle is tangent to all these surrounding circles. What fraction of the area of the big circle is covered by the small circles?

3. Eight people go to a restaurant and each orders some different dish for dinner. The waiter is inexperienced. He forgot who ordered what and does not recognize any of the dishes, so he can't ask his patrons. He starts to serve the dishes randomly. If you are served third and you see the two people who got served before you frown (but you can't see the dish they got) what is the probability that you will **NOT** receive the dish you ordered?

4. A three dimensional figure is a pyramid whose height is 10 and whose base is an ellipse with major and minor axes of lengths 6 and 18. What is its volume?

5. Consider the circle whose equation is $x^2 - 8x + y^2 - 8y = -28$ find the point on this circle which is closest to the origin.

6. Given the following information about a sequence:

$$a_0 = 1 \quad a_1 = 4 \qquad a_n = 5a_{n-1} - 6a_{n-2}$$

find an explicit formula for a_n

7. Observe the following

$$1 * 2 = 1 * 2 * 3/3$$

$$1 * 2 + 2 * 3 = 2 * 3 * 4/3$$

$$1 * 2 + 2 * 3 + 3 * 4 = 3 * 4 * 5/3$$

What is an expression for $1 * 2 + 2 * 3 + \ldots + n(n+1)$??

8. Consider the parabolas described by the equation

$$y + Ax^2 + Bx + C = 0$$

where A and B are fixed. Find a value for C so that the vertex is on the x-axis.

9. An integer is a *perfect number* if it is equal to the sum of its divisors (where the divisors include 1 but not the number itself.) 6 is such a number, what is another?

10 On a sphere of radius 2 is a triangle formed by two lines of longitude from the north pole to the equator with an angle at the north pole of 30 degrees, and a third line along the equator. What is the area of this triangle?

11 Which of the following is a factor of $ab(c^2+d^2)+cd(a^2+b^2)$?

a)ab b) c^2+d^2 c) $ab+cd$ d) $ac+bd$

12 An 8' by 10' table sits in the corner of a square room, as shown in Fig. 1 below. The owners of the table wish to move it to the position shown in Fig. 2 without tilting it or taking it apart. What is the smallest integer value for S so that this is possible?

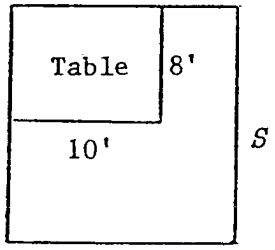

Figure 1

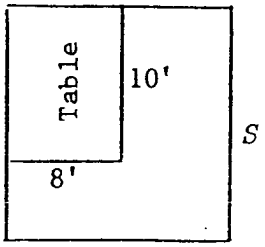

Figure 2

13 What is the simplest value for the product

$$(1-1/2^2)(1-1/3^2)(1-1/4^2)...(1-1/9^2)(1-1/10^2)?$$

14 What is the *reflection* of the point $(1,5)$ about the line $y=x/2$?

SCHOOL Answer key

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QUESTION	ANSWER	SCORE	QUESTION	ANSWER	SCORE
1	$\boxed{4}$		8	$\boxed{C = B^2/4A}$	
2	$\boxed{7/9}$		9	$\boxed{28 \text{ or } 496 \text{ or } 8128}$	
3	$\boxed{\frac{227}{258}}$		10	$\boxed{2\pi/3}$	
4	$\boxed{270\pi}$		11	$\boxed{d}$	
5	$\boxed{(4-\sqrt{2}, 4-\sqrt{2})}$		12	$\boxed{\sqrt{164} \text{ so } 13}$	
6	$\boxed{a_n = -2^n + 2 * 3^n}$		13	$\boxed{11/20}$	
7	$\boxed{\frac{n(n+1)(n+2)}{3}}$		14	$\boxed{(23/5, -11/5)}$	

Team Total

# CORRECT × 3 =	
Individual Totals	