DIRECTIONS: Please write your answers on the Team Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators MAY NOT be used.

1. Given two concentric circles of radius r and R, $r < R$, such that a *40 in.* chord of the larger circle both subtends an angle of *120°* and is tangent to the smaller circle, find r.

2. Using the diagram below, select all of the logical statements which express the shaded area:
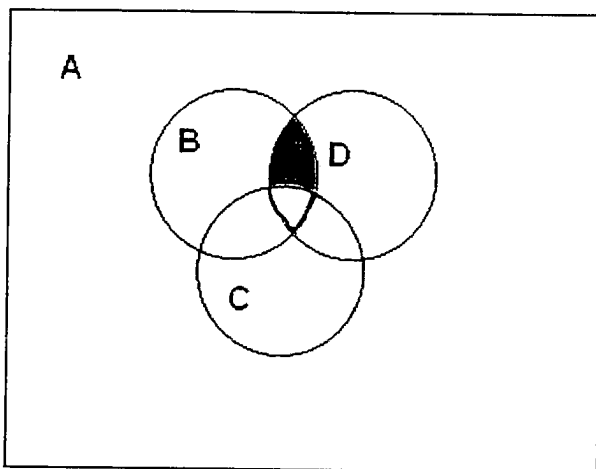
   a. $B \cap D \cap \overline{C}$

   b. $\overline{B} \cup \overline{D} \cup C$

   c. $\overline{\overline{B} \cap \overline{D} \cup C}$

   d. $\overline{B \cap D \cap C}$

   e. $\overline{\overline{B} \cap \overline{D} \cup C}$

   f. $\overline{\overline{B \cap D} \cap C}$

   g. $\overline{C} \cap A \cap D \cap B$

3. Given that $\lfloor n \rfloor$ is the floor function equal to the greatest integer less than or equal to n, evaluate $\displaystyle\sum_{N=1}^{1024} \lfloor \log_2 N \rfloor$.

4. A point D on the interior of an equilateral triangle $\triangle ABC$ defines an isosceles triangle $\triangle DBC$ such that the area of $\triangle ABC$ is twice that of $\triangle DBC$. What is the length of $\overline{AD}$ in terms of the length of $\overline{AC}$?

5. What is $6543.21_{12} - 6543.21_{10}$ in base 10?

6. An ellipse is defined by $\frac{x^2}{75} + \frac{y^2}{100} = 1$. A ray through *(0, 5)* reflects off of the ellipse at *(7.5, -5)*. Where does the reflected ray cross the *y-axis*?

7. If ω is an acute angle and $\sin\left(\frac{\omega}{2}\right) = \sqrt{\frac{x-1}{2x}}$, then express *tan(ω)* in terms of *x*?

8. If it is true for a triangle with side lengths *a*, *b* and *c* that *(a + b + c)(a + b − c) = 3ab*, then what is the measure of the triangle's angle opposite the side of length *c*?

9. If x satisfies $0 < x < 1$, arrange x, x^x and $x^{x^{x}}$ in *increasing* order of magnitude.

10. Evaluate $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}^{16}$.

11. Use the information given to calculate the area of the small highlighted triangle.

2 cm.
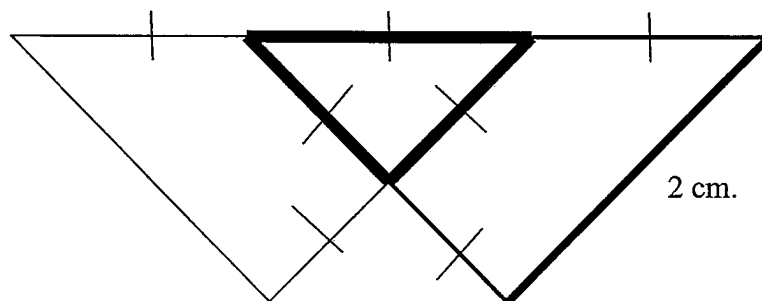
12. f(x) is a cubic polynomial which passes through (-3,-208), (-1,-32),(1,-24) and (3,8). What is f(2)?

13. An aquarium partially filled with water has two parallel rectangular faces 8" wide and 10" high, and the other two sides are rectangles 10" high but of unknown length. When the aquarium is tilted so that the water just covers an entire 8"x10" side, it covers just ¾ of the rectangular bottom. What is the depth of the water in the tank when it is level?

14. What are the solutions to $2^{2x^2-7x+6} = 1$?

SCHOOL **Answer Key**

WORCESTER POLYTECHNIC INSTITUTE

EIGHTEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 19, 2005
TEAM EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE	QUESTION	ANSWER	SCORE
1	$r = \frac{20\sqrt{3}}{3}$		8	60° or $\frac{\pi}{3}$ **radians**	
2	a, e, g		9	x, $x^{x^{x}}$, and x^{x}	
3	8204		10	$\begin{bmatrix} \frac{1}{65536} & 0 \\ 0 & \frac{1}{65536} \end{bmatrix}$	
4	$AD = \frac{\sqrt{3}}{4} AC$		11	$\frac{\sqrt{3}}{4}$ cm^2	
5	$4595\frac{3469}{3600}$ **or** $4595.9636\overline{1}$		12	f(2) = -23	
6	*(0 , -5)*		13	$^{15}/_{4}$"	
7	$\sqrt{x^2 - 1}$		14	x = 1.5 and x = 2	

Team Total

# CORRECT × 3 =	
Individual Totals	