WORCESTER POLYTECHNIC INSTITUTE
NINETEENTH ANNUAL INVITATIONAL MATH MEET
October 18, 2006
TEAM EXAM QUESTION SHEET

1. In Pascal's triangle, for the first **n** rows, the quotient of the number of integers which are not 1 to the number of integers that are 1 is what?

2. If $\log_2(\log_3(\log_4(x))) = \log_3(\log_4(\log_2(y))) = \log_4(\log_2(\log_3(z))) = 0$
   then what is the sum of $x + y + z$?

3. What is the value of the real number x such that 64^{x-1} divided by 4^{x-1} is 256^{2x} ?

4. The square of an integer is called a *perfect square*. If **x** is a perfect square, what is the next largest perfect square?

5. If **r** is positive and the line $x + y = r$ is tangent to the curve $x^2 + y^2 = r$ then what is the value of **r**?

6. Assume that **p** is a positive integer such that $2^p - 1$ is prime. Consider the number $n = 2^{p-1}(2^p - 1)$. What is the *sum* of all divisors of **n**? (*not* including **n** itself)

7. Find the smallest positive integer such that

$$\sqrt{n} - \sqrt{n-1} < .01$$

8. A semicircle of radius 4 is formed into a cone by attaching the two radii together. What is the volume of the resulting cone?

9. Find a value for **b** so that the following system has solutions

$$x + 2y + 2z = 1$$
$$x - y + 3z = 3$$
$$x + 11y - z = b$$

10. A triangle in R^3 is determined by the points $Q_1:(2,1,7)$, $Q_2:(5,x,7)$ and $Q_3:(5,5,19)$. Find values for x so that the triangle $Q_1Q_2Q_3$ is a right triangle.

11. The sum of all but one of the interior angles of a convex polygon is **2570°**. What is the size of the remaining angle?

12. A number is selected at random from the set of natural numbers. What is the probability that the number is divisible by 3, not divisible by 5 and divisible by either 4 or 6?

13. Where are the foci for the conic section described by the following equation?

$$9x^2 - 54x + 16y^2 + 160y + 337 = 0$$

14. It is possible to place positive integers into the 21 vacant squares into the 5x5 square shown below so that the numbers in each row and each column form arithmetic sequences. Find the number that must occupy the empty square marked by an **X**.

			X	
	74			
				186
		103		
0				

SCHOOL_____

WORCESTER POLYTECHNIC INSTITUTE

NINETEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 18, 2006
TEAM EXAM **ANSWER** SHEET

QUESTION	ANSWER	SCORE		QUESTION	ANSWER	SCORE
1	$(n^2 - 3n + 2)/(4n-2)$			**8**	$8\pi\sqrt{3}/3$ or $8\pi/\sqrt{3}$	
2	89			**9**	$b = -5$	
3	$x = -1/3$			**10**	$x = 1,5,-5/4, 41$	
4	$x + 2\sqrt{x} + 1$			**11**	$130°$	
5	$r = 2$			**12**	$2/15$	
6	n			**13**	$(3 +/- \sqrt{7}, -5)$	
7	2501			**14**	142	

Team Total

# CORRECT × 3 =	
Individual Totals	