WORCESTER POLYTECHNIC INSTITUTE
TWENTIETH ANNUAL INVITATIONAL MATH MEET
OCTOBER 17, 2007
INDIVIDUAL EXAM QUESTION SHEET WITH ANSWERS

1. What is the 2007th digit in the decimal representation of 1/14?

Ans: 1

2. Examining the graph of $f(t) = e^{2t}\sin(\pi t)$ for $t \geq 0$ where t is in seconds, in the first 4 minutes, how many times does the graph cross the t axis?

Ans: 239

3. If $y^d y^e = 1$ and y is not +/-1, evaluate $4d - e^2 + d^2 + 4e - 10.$

Ans: -10

4. It is known that the sum of the first n odd integers is n^2. Determine the simplest form of

$$\frac{1 + 3 + 5 + ... + 739}{741 + ... + 1479}$$

Ans: 1/3

5. A triangle has sides $5\sqrt{3}$, $10\sqrt{3}$ and **15.** Find the length of the bisector of the second largest angle.

Ans: 10

6. A solar reflector is made of 36 triangular sections with sides of 6.2 mm, 6.2 mm and 1.1 mm. What is the total area of the reflector rounded to the nearest square mm?

Ans: 122 mm^2

7. How many permutations of the digits **0,1,2,...,9** have an even digit in the first place and **1,2,3,4** or **5** in the last place?

Ans: (2*4 + 3*5)* 8! = 927360

8. The **nth** Mersenne Number is given by $M_n = 2^n - 1$ where **n** is a positive integer. What is the binary form of M_n ?

Ans: 111...1 (n times)

9. For what real values of **x** does the following equation hold?

$$\sqrt{x + 2\sqrt{x-1}} + 4\sqrt{x - 2\sqrt{x-1}} = 2\sqrt{x-1} + 12$$

Ans: x = 26

10. In the expansion of $(a - 2b)^{11}$ what is the coefficient of $a^8 b^3$?

Ans: -1320

11. A parabola has its focus at **(2,6)** and directrix the line **y = 10.** What is its equation?

Ans: $y - 8 = \dfrac{-(x-2)^2}{8}$