WORCESTER POLYTECHNIC INSTITUTE
TWENTIETH ANNUAL INVITATIONAL MATH MEET
OCTOBER 17, 2007
TEAM EXAM QUESTION SHEET WITH ANSWERS

1. A ball was floating in a lake when the lake froze. The ball was removed (without breaking the ice), leaving a hole 24 cm across the top and 8 cm deep. What was the radius of the ball in centimeters?

Ans: 13 cm

2. Point of tangency of two spheres described by

$$x^2 + y^2 + z^2 = 121$$
$$(x-4)^2 + (y-12)^2 + (z-18)^2 = 121$$

Ans: (2,6,9)

3. Consider the graphs of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where **A** is a *positive* constant and **x** and **y** are real variables. In how many points do the two graphs intersect?

Ans: 2

4. A certain 5 digit number has the property that if a 1 is placed after it, it is 3 times as large as with a 1 placed before it. What is that number?

Ans: 42857

5. We have two concentric circles and wish to find the area of the annulus between them. If we draw a chord through the outer circle tangent to the inner circle, its length is 20 inches. What is that area?

Ans: 100Pi

6. Factor the following polynomial over the reals as completely as possible:

$$x^7 - \frac{5}{2}x^6 + \frac{15}{2}x^5 - 13x^4 - 23x^3 + \frac{177}{2}x^2 - \frac{171}{2}x + 27$$

Ans: $(x-3/2)(x+2)(x^2+9)(x-1)^3$

7. If $z = \sqrt{2} + \sqrt{2}\,i$, what is z^{20} ?? (where $i = \sqrt{-1}$)

Ans: z = -1048576

8. Simplify $\mathbf{(a + b)^{15} \ mod\ 15}$

ans: $a^{15} + 5\,b^3\,a^{12} + 3\,b^5\,a^{10} + 10\,b^6\,a^9 + 10\,b^9\,a^6 + 3\,b^{10}\,a^5 + 5\,b^{12}\,a^3 + b^{15}$

9. Simplify $\mathbf{3^{218}\ mod\ 7}$.

Ans: 2

10. Determine the file size in Megabytes (Mb) for a digital recording made with samples of size 2 bytes taken 44,100 times per second, in stereo, for 40 minutes. Your answer should be rounded to the nearest tenth of a Mb.

Ans: 423.4 Mb

11. Find the sum **17 + 22 + 27+. . . +182**

Ans: 3383

12. If it is given that

$\mathbf{\log_x w=24 \quad \log_y w =40 \quad \log_{xyz} w = 12}$

then what is $\mathbf{\log_z w}$? **(x, y** and **z** are all positive numbers**)**

Ans: 60

13. Simplify the following to a single fraction

-5/4 + 5/8 – 5/16 + 5/32 . . . - 5/1024

Ans: -855/1024

14. Determine $\begin{pmatrix} 7/5 & -4/5 \\ 6/5 & -7/5 \end{pmatrix}^{p}$ where $\boldsymbol{p}$ is prime and positive**.**

Ans: same matrix

(true for odd powers therefore prime**)**