WORCESTER POLYTECHNIC INSTITUTE
TWENTY-FIRST ANNUAL INVITATIONAL MATH MEET
OCTOBER 21, 2008
INDIVIDUAL EXAM QUESTION SHEET WITH ANSWERS

1. Consider the conic $\mathbf{x^2/9 + y^2/16 = 1}$ and the line $\mathbf{3y - 4x = 12}$. At what points, if any, do they meet?

Ans: (-3,0 and (0,4) 1 point

2. A Double Mersenne Number, $\mathbf{D_n}$, is of the form $\mathbf{4^n - 1}$ where $\mathbf{n}$ is a positive integer. What is the binary form (base 2) for such a number?

Ans: 2n 1's 1 point

3. The symbol $\mathbf{25_b}$ represents a 2 digit number to the base $\mathbf{b}$. If the number $\mathbf{52_b}$ is twice $\mathbf{25_b}$ then what is the value of $\mathbf{b}$?

Ans: b = 8 1 point

4. What is the number of digits in the number $\mathbf{2^{12}*5^8}$?

Ans: 10 1 point

5. A parabola is known to have its vertex at $\mathbf{(2, 5)}$ and its focus $\mathbf{2}$ units to the left of the vertex. What is its equation?

Ans: $x-2 = -1/(4*2)\ (y-5)^2$ 2 points

6. Two concentric circles are formed. What must be the ratio of the larger to smaller radii so that the area in between them is 84% of the area of the larger circle?

Ans: 2.5 : 1 ratio 2 points

7. Express as single complex number: $\mathbf{1 + i + i^2 + i^3 + \ldots + i^{100}}$ where $\mathbf{i^2 = -1}$

Ans: 1 2 points

8. How many different 5 digit numbers can be constructed using the digits 1,1,1,4,7?

Ans: 20 2 points

9. The parabola $\mathbf{y = ax^2 + bx + c}$ has vertex **(p,p)** and **y** intercept at **(0, -p)** where $\mathbf{p \neq 0}$. What must the value of **b** equal in order for this to happen?

Ans: b = 4 3 points

10. A fair six-sided die is tossed three times and the resulting sequence of numbers is recorded. What is the probability of the event **E** that either all of the numbers are equal or none of them is a **4**?

Ans: p(E) = 7/12 3 points

11. Consider the solutions to the equation $\mathbf{3x^2 - 4x + k = 0}$. The value of **k** for which the product of the roots is a maximum is what?

Ans: k = 4/3 3 points