1. Suppose \( a \) and \( b \) are digits (integers from 0 through 9). What number divides \( abba \) for all choices of \( a \) and \( b \)?

   Ans: 11

2. Find all points \((x,y)\) in the Euclidean plane satisfying \( F(x,y) = 0 \) where

\[
F(x,y) = xy^4 - y^4 + x^3 y^2 - x^2 y^2 - x^3 + x^2 - x + 1
\]

   Ans: union of the lines: \( x = 1, y = 1 \) and \( y = -1 \).

3. Find \( x > 0 \) satisfying the following equation

\[
(4x)\log_4(4) = (5x)\log_5(5)
\]

   Ans: 1/20

4. Arithmetic series sum: \( 200 + 205 + 210 + 215 + \ldots + 2745 \)

   Ans: 750975

5. Factor as completely as possible over the reals the following polynomial:

\[
x^5 + x^4 + 4x^3 - x^2 - 19x + 14
\]

   Ans: \((x-1)^2(x+2)(x^2 + x + 7)\)

6. A right circular cone is formed by cutting up a circular piece of paper which is 10 units in diameter. A sector of angular measure \( 2\pi/3 \) radians is removed and the remaining paper formed into the cone. What is its volume?

   Ans: \( \pi \left( \frac{10}{3} \right)^2 \sqrt{25 - \left( \frac{10}{3} \right)^2} \)
7. Hoodsie the cow is tethered to a corner of a barn which is 20’ by 40’. Her rope is 60’ long. How much grazing area does she have? No barn doors are open.

\[ \text{Ans: } 3200 \pi \]

8. How many zeroes are at the end of the expansion of \( 31! \)?

\[ \text{Ans: 7} \]

9. A circle inscribed in an equilateral triangle and a square inscribed in the circle. What is the ratio of the area of the triangle to that of the square?

\[ \text{Ans: } \frac{3 \sqrt{3}}{2} \]

10. What is the minimum value of \( \sqrt{x^2 + y^2} \) if \( 5x + 12y = 60 \)?

\[ \text{Ans: } \frac{60}{13} \]

11. The sum of an infinite geometric series with \( -1 < r < 1 \) as its common ratio, is 15. The sum of the squares of the terms in this series is 45. What is the first term in this series?

\[ \text{Ans: } a = 5 \]

12. How many permutations of the letters D,O,R,E,M,I do not contain the word DO, RE, MI; that is none of the words DO, RE and MI appears as consecutive letters?

\[ \text{Ans: 426} \]

13. For some real number \( r \), the polynomial \( 8x^3 + 4x^2 - 42x - 45 \) is divisible by \( (x - r)^3 \). What is \( r \)?

\[ \text{Ans: } \frac{3}{2} \]

14. How many real solutions are there to the equation: \( x^{256} - 256^{32} = 0 \)?

\[ \text{Ans: 2} \]