WORCESTER POLYTECHNIC INSTITUTE
TWENTY-FIRST ANNUAL INVITATIONAL MATH MEET
OCTOBER 21, 2008
TEAM EXAM QUESTION SHEET WITH ANSWERS

1. Suppose **a** and **b** are digits (integers from 0 through 9). What number divides **abba** for all choices of **a** and **b**?

Ans: 11

2. Find all points **(x,y)** in the Euclidean plane satisfying **F(x,y) = 0** where

$$F(x,y) = xy^4 - y^4 + x^3y^2 - x^2y^2 - x^3 + x^2 - x + 1$$

Ans: union of the lines: x = 1, y = 1 and y= -1.

3. Find **x > 0** satisfying the following equation

$$(4x)^{\log_6(4)} = (5x)^{\log_6(5)}$$

Ans: 1/20

4. Arithmetic series sum: **200 + 205 + 210 + 215 + . . . 2745**

Ans: 750975

5. Factor as completely as possible over the reals the following polynomial:

$$x^5 + x^4 + 4x^3 - x^2 - 19x + 14$$

Ans: $(x-1)^2(x+2)(x^2 + x + 7)$

6. A right circular cone is formed by cutting up a circular piece of paper which is 10 units in diameter. A sector of angular measure **2π/3** radians is removed and the remaining paper formed into the cone. What is its **volume**?

Ans: $\frac{\pi}{3}\left(\frac{10}{3}\right)^2\sqrt{25-\left(\frac{10}{3}\right)^2}$

7. Hoodsie the cow is tethered to a corner of a barn which is 20′ by 40′. Her rope is 60′ long. How much grazing **area** does she have? No barn doors are open.

Ans: 3200Pi

8. How many zeroes are at the end of the expansion of **31**!

Ans: 7

9. A circle inscribed in an equilateral triangle and a square inscribed in the circle. What is the ratio of the area of the triangle to that of the square?

Ans: $3\sqrt{3}$:2

10. What is the minimum value of $\sqrt{x^2 + y^2}$ if **5x + 12y = 60**?

Ans: 60/13

11. The sum of an infinite geometric series with **-1< r < 1** as its common ratio, is **15**. The sum of the squares of the terms in this series is 45. What is the first term in this series?

Ans: a = 5

12. How many permutations of the letters D,O,R,E,M,I do not contain the word DO, RE, MI; that is none of the words DO, RE and MI appears as consecutive letters?

Ans: 426

13. For some real number **r**, the polynomial $\mathbf{8x^3 + 4x^2 - 42x - 45}$ is divisible by $\mathbf{(x - r)^2}$. What is **r** ?

Ans: 3/2

14. How many real solutions are there to the equation: $\mathbf{x^{256} - 256^{32} = 0}$?

Ans: 2