

Worcester Polytechnic Institute Twenty-Second Invitational Math Meet
October 21, 2009-Individual Exam

1. What is the slope of any line perpendicular to the line satisfying the following equation?

$$\frac{x-8}{9-y} = \frac{4}{3}$$

ans: 4/3

2. The area of a circular region is numerically equal to a third of its circumference. What is its radius?

ans: $r = 2/3$

3. What is

$$\sum_{r=0}^{24} \sum_{s=1}^{40} \left(\frac{1}{s} - \frac{1}{s+1} \right) ?$$

ans: (25)(1 - 1/41) or 1000/41

4. If $f(x) = \frac{2x-1}{3x+5}$ find a formula for f^{-1} ans: $f^{-1}(y) = \frac{5y+1}{-3y+2} = \frac{-5y-1}{3y-2}$

5. Find all real roots of $x^4 - 8x^3 + 22x^2 - 24x + 9$ ans: $x = 1, 1, 3, 3$

6. Two card players cut a deck of 52 cards three times. What is the probability that they cut two cards of the same face value, repeat this (with a possibly different face value), and then cut two cards of different face value, in that order?

$$\text{ans: } \left(\frac{4}{52} \right)^2 \frac{48}{52}$$

7. Find the point on the line $x + 2y = 20$ which is closest to the point (1,-3)

ans: (6,7) or $x=6$ $y=7$

8. If the y axis is reflected about the line $y = \sqrt{3}x$ what is the equation of the resulting line?

$$\text{ans: } y = \frac{1}{\sqrt{3}}x$$

9. What is the probability there is at least one pair of students in a class of 40 who share the same birthday? Assume it is not a leap year. ans: $1 - \frac{365!}{365^{40}(365-40)!} \approx .891$

10. An $n \times n$ matrix B is of the form PQP^{-1} where Q is a diagonal matrix whose diagonal is a binary number with n bits. What is the matrix B^p where p is a prime number? ans: B

11. If

$$\log_8(\log_3(\log_2(x))) = 0 \text{ then } x^{1/3} \text{ is equal to what? ans: } x^{1/3} = 2$$