

Worcester Polytechnic Institute Twenty-Second Invitational Math Meet  
October 21, 2009-Team Exam

1. Solve for  $x$ :  $x^3 - [x] = 3$  where  $[x]$  denotes the greatest integer less than or equal to  $x$ .

ans:  $4^{1/3}$

2. Assume the set  $M$  is all ordered pairs of integers. Define a multiplication on  $M$ , #, by the following formula

$$(x,y) \# (z,w) = (xz + 2yw, xw + yz)$$

What is the identity element for the operation # on  $M$ ? ans:  $(1,0)$

3. The roots of  $px^2 + qx + 1$  are  $1/2$  and  $1/3$  What are the roots of  $qx^2 - px - 1$ ?

ans:  $-1$  and  $-1/5$

Express  $\sum_{k=-2}^{+3} \frac{1}{8^k}$  as a binary number ans:  $1001001.001001001$

4. For a point  $(x,y)$  on the curve  $4x^2 - 16x + 9y^2 + 18y - 10 = 1$

what is the sum of the distances from it to the points  $(2 + \sqrt{5}, -1)$  and  $(2 - \sqrt{5}, -1)$ ?

ans:  $6$

5. In a large on-line discussion group, the average age of all males was 19 while the average of all females was 15. If the average age of all members was  $17 \frac{1}{2}$  years, what was the ratio of females to males? ans:  $3:5$

What is the 76<sup>th</sup> natural number not divisible by 7 or 2? ans:  $177$

6. Solve for  $x > 0$   $2 \log_4(x) = \log_4(4) + \log_4(4x - 12)$  ans:  $x = 4, 12$

7. Solve for  $x$  between 0 and  $\pi$

$$4\sin^4(x) = 4\cos^2(x) - 1$$

ans:  $\pi/4$  and  $3\pi/4$

8. Consider the region in the first quadrant of the  $(x,y)$  plane defined by

$$x \leq 15, \quad y \leq 15 \quad 3x + 2y \leq 57$$

Find the point where the function  $P(x,y) = x + 6y$  is the greatest

ans:  $x = 15$   $y = 9$  or  $(15,9)$

9. What is the coefficient of  $x^4y^2z$  in the expansion of  $(x + 2y + z)^7$ ? ans:  $420$

10. Compute the value of  $(1 - \frac{3}{20})(1 - \frac{3}{19})(1 - \frac{3}{18}) \dots (1 - \frac{3}{8})(1 - \frac{3}{7})$  ans:  $1/57$

11. How many zeroes are at the end of the expansion of  $30!$  ans:  $7$

12. A unit of currency is called a HAG in the Land of Pyt is a rectangular bill 24 Oras by 32 Oras in size. A person takes a bill and folds it diagonally. What is the area of the resulting pentagon?

ans:  $468 \text{ Oras}^2$