WORCESTER POLYTECHNIC INSTITUTE
TWENTY-THIRD INVITATIONAL MATH MEET
OCTOBER 20, 2010
INDIVIDUAL EXAM QUESTION SHEET WITH ANSWERS

DIRECTIONS: Please write your answers on the **Individual Answer Sheet** provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators **MAY NOT** be used.

1. The straight line $ax + 85y = 1$, where a is an integer, passes through the point (71,230). What is the greatest common divisor of a and 85?

   Ans: 1

2. Evaluate $10^{\log_{100} 9}$ **Ans:** 3

3. The line $y = 2x$ is the perpendicular bisector of the segment AB where A has coordinates (-3,-1). What are the coordinates of B? **Ans:** (1, -3)

4. In a certain high school there are 605 students. There are 40 more sophomores than freshman and one half as many freshmen as juniors. The number of seniors is 30 less than 3 times the number of freshman. How many seniors are there?

   Ans: 225

5. Suppose f: R -> R is defined by $f(y) = \sqrt{3y^2 - 3y - 60}$. What is its *domain*?

   Ans: $y \geq 5$ and $y \leq -4$

6. A triangle has sides of $12ryz$, $6ry^2\text{-}6rz^2$ and $6ry^2 + 6rz^2$. What is the largest of its three angles?

   Ans: 90°

7. What are the cube roots of -27?

   Ans : -3, 3/2 +/- 3√3/2 i

8. A triangle is formed on a globe of radius 5, by: the equator, the line at 0˚ Longitude and the line at 30˚ East Longitude. What is its area?

Ans: $25\pi/6$

9. What is the 39^{th} power of the following matrix: $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$

Ans: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

10. A function f, defined for all real non-zero numbers x, satisfies $3f(x) + 4f(1/x) = 5x$. Find all x such that $f(x) - f(-x) = 0$

Ans: $\frac{2\sqrt{3}}{3}$

11. Suppose x, y, z and n are nonnegative integers such that $x^n + y^n = z^n$. What can be said about n?

Ans: n=1 or n=2 (FLT)