WORCESTER POLYTECHNIC INSTITUTE
TWENTY-THIRD ANNUAL INVITATIONAL MATH MEET
OCTOBER 20, 2010
TEAM EXAM QUESTION SHEET WITH ANSWERS

DIRECTIONS: Please write your answers on the **Team Answer Sheet** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators **MAY NOT** be used.

1. Consider the cone $z^2 = (x^2 + y^2)/3$ and suppose it is cut by the plane $z = x + 5$. What geometric object results?

   Ans: hyperbola

2. An ellipse has its foci on the x axis at $x = 2$ and $x = -2$ and crosses the x axis at x=-5. What is its area?

   Ans: $5\sqrt{21}\,\pi$

3. What is the base 3 form of the number $3^{10} - 1$?

   Ans: $222\ldots2_3$ (10 2's in a row)

4. Find a value of the parameter k so the system has no solution:

   $$x + y + kz = 1$$
   $$x + ky + z = 4$$
   $$kx + y + z = 7$$

   Ans: k = 1 or -2

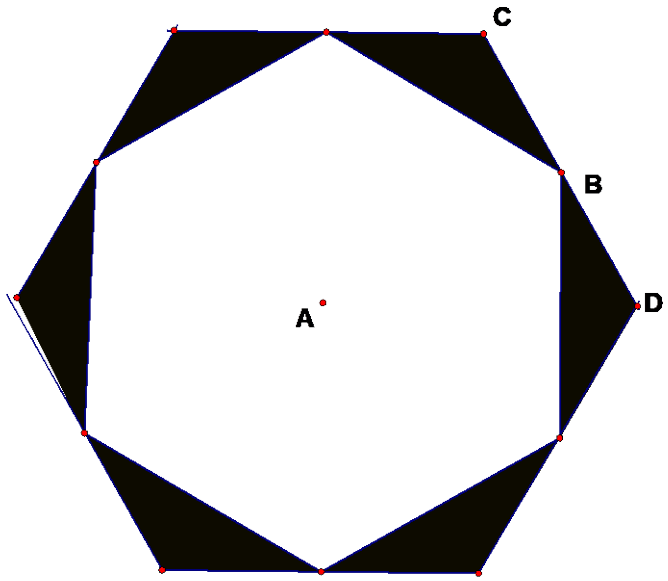
5. In the expression $(2x - 5y)^7$ what is the coefficient of x^5y^2 ?

   Ans: 16800

6. What is the sum of $100 + 107 + 114 + \ldots + 373$?

   Ans: 9460

7. Given 2 regular hexagons each with its center at A, also AB = 1 and AB $\perp$ CD at B. Find the area of the shaded region.



Ans: sqrt(3) /2

8. Water flowing from two pipes into a vat can fill the vat in 2 hours. It takes the smaller pipe alone 3 hours longer than the larger pipe to fill the vat. One day both pipes were opened simultaneously into the empty vat. At the end of two hours it was discovered that one pipe had become clogged and the vat was only half full. How long was that pipe clogged?

Ans: ½ hour

9. One root of $2x^3 - 6x^2 + px + q = 0$ is $x = -2$. The other two roots are equal. Find the value of q.

Ans : 25

10. Consider the region in the first octant beneath the plane $3x + 4y + 6z = 24$. What is its volume?

Ans: 32

11. Factor completely: $a^2 + 6b^2 - 12c^2 - ac - bc - 5ab$

Ans: (a -2*b* + 3*c*)(a - 3*b* - 4*c*)

12. Solve for *x*:

$$\log_{27}(x) + 4\log_x(27) = \log_2(9) * \log_3(4)$$

Ans: $x = 27^2$ or 729

13. One cube root of a real number is $1 - \sqrt{3}\,i$ (where $i = \sqrt{-1}$). What are the other two roots?

Ans: $1 + \sqrt{3}\,i$ and - 2

14. A radioactive isotope is known to decay exponentially and have a half life of 40 days. If there are originally 1408 grams, how long will it take for there to be 22 grams left?

Ans: 240 days