

**WORCESTER POLYTECHNIC INSTITUTE**  
**TWENTY-FOURTH INVITATIONAL MATH MEET**  
**OCTOBER 19, 2011**  
**INDIVIDUAL EXAM QUESTION SHEET WITH ANSWERS**

DIRECTIONS: Please write your answers on the **Individual Answer Sheet** provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators **MAY NOT** be used

**Part A (1-4)**

1. If the convex area bounded by the  $x$ -axis, the line  $y = mx + 4$ ,  $x = 1$  and  $x = 4$  equals 7, then what is  $m$ ?

Ans:  $m = -2/3$

2. If  $f(x) = ax^7 + bx^3 + cx - 5$  and  $f(-7) = 7$  then  $f(7) = ?$

Ans: -17

3. Three points determine a triangle. They are  $(2,1,7)$   $(9,3,6)$  and  $(1,1,2)$ . What is the area of it?

Ans:  $5\sqrt{14}$

4. Consider the points  $(7, 1, 5)$ ,  $(9, 3, 2)$  and  $(13, m, -4)$ . Determine  $m$  so they are colinear.

Ans:  $m=7$

**Part B (5-8)**

5. What is the remainder when  $x^3 - 2$  is divided by  $x^2 - 2$ ?

Ans:  $2x - 2$

6. A vertical line divides the triangle whose vertices are  $(0, 0)$   $(1, 1)$  and  $(0, 1)$  into two regions of equal area. What is its equation?

Ans:  $x = 1 - \frac{\sqrt{2}}{2}$

7. If the length of the diagonal of a cube is  $a$ , then what is the surface area of the cube?

Ans:  $2a^2$

8. Determine  $z = \sum_{k=0}^{10} c(10, k)$  where  $c(n, k)$  is the number of ways to choose  $k$  from  $n$ .

Ans:  $z = 1024$

**Part C (9-11)**

9. If  $\frac{\log a}{p} = \frac{\log b}{q} = \frac{\log c}{r} = \log x$  all logs to the same base,  $x \neq 1$  and further, if

$$\frac{b^2}{ac} = x^y$$

then what is  $y$ ?

Ans:  $2q - p - r$

10. What is the sum of the squares of all real numbers satisfying

$$x^{256} - 256^{32} = 0$$

Ans: 8

11. Consider a parabola with focus at  $(0, 2)$  and directrix  $y = -2$ . What is an equation for the tangent line to it at the point where  $x = +4$  ?

Ans:  $y = x - 2$