1. A circle is inscribed in a square of side \( m \), then a square inside that circle, then a circle inside the latter square and so on. If \( S_n \) is the sum of the areas of the first \( n \) such circles so inscribed, then as \( n \) grows without bound, what does \( S_n \) approach?

Ans: \( \frac{\pi m^2}{2} \)

2. If \( S = 1! + 2! \cdots + 99! \) then what is the units value in \( S \)?

Ans: 3

3. The number \( 2^{48} - 1 \) is exactly divisible by two numbers between 60 and 70. What are they?

Ans: 63, 65

4. If \( z = \frac{1}{2} + \frac{\sqrt{3}}{2} i \) in Cartesian form, find \( z^{15} \).

Ans: -1

5. If \( f(x) = \log \left( \frac{1+x}{1-x} \right) \) then what is \( f\left( \frac{3x+x^3}{1+3x^2} \right) \) in terms of \( f(x) \)?

Ans: \( 3f(x) \)

6. The sum of all but one interior angles of a convex polygon is \( 2570^\circ \). The remaining angle must be what?

Ans: \( 130^\circ \)

7. The number of real solutions to

\[
\frac{x}{100} = \sin(X)
\]

is what?

Ans: 63
8. In what follows, all matrices are \( n \times n \). Given \( A = QRS \) where \( Q = S^{-1} \) and the entries of \( R \) satisfy \( r_{ij} = \begin{cases} 0, & i \neq j \\ (-1)^3i, & i = j \end{cases} \). What is \( A^{2p+1} \) where \( p \) is any prime number?

Ans: \( A \)

9. The greatest integer that will divide the three integers \( 13, 511 \), \( 13,903 \) and \( 14, 589 \) and leave the same remainder is ________.

Ans: 98

10. If the graph of \( x^2 + y^2 = m \), where \( m > 0 \), is tangent to that of \( x + y = k \) what must \( k \) be?

Ans: \( \pm \sqrt{2m} \)

11. A number \( n \) has 3 digits when expressed in base 7. When \( n \) is expressed in base 9 the digits are reversed. What is the middle digit?

Ans: 0

12. Let \( S = 2 + 4 + 6 + \cdots + 2N = 2 \left( 1 + \cdots + N \right) \) where \( N \) is the smallest positive integer such that \( S > 1 \) million. Then the sum of the digits of \( N \) is what?

Ans: 1 (since \( N = 1000 \))

13. Let \( n \) be the number of ways that \$10 \) can be changed into dimes and quarters, with at least one of each being used. Then what is \( n \)?

Ans: 19

14. Find the 6\(^{th} \) root of \( -729 \) which lies in the third quadrant of the complex plane.

Ans: \( \frac{-3\sqrt{3}}{2} - \frac{3}{2} i \)