

**WORCESTER POLYTECHNIC INSTITUTE
TWENTY-FIFTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 17, 2012
TEAM EXAM QUESTION SHEET WITH ANSWERS**

DIRECTIONS: Please write your answers on the **Team Answer Sheet** provided. This part of the contest is 45 minutes. All 14 problems are counted equally. Calculators **MAY NOT** be used.

1. What is the value of $\cos \frac{\pi}{8} + \cos \frac{3\pi}{8} + \cos \frac{5\pi}{8} + \cos \frac{7\pi}{8}$?

Ans: 0

2. A class has four girls and four boys. These students line up at random, one after another. What is the probability that no boy is right next to another boy, and no girl is right next to another girl?

Ans: $\frac{1}{35}$

3. Pick any natural number, double it and add 5. Double this answer and add two. Now subtract the number you picked. The result will always be a multiple of what integer ?

Ans: 3

4. A box contains ten 40 cent stamps, three 30 cent stamps and two 20 cent stamps. Four stamps are drawn at random. What is the probability their combined value does not exceed \$1.30?

Ans: $420/1365 = 4/13$

5. How many digits are in the base 10 representation of the number $4^{42} 5^{80}$?

Ans: 82 digits

6. Pencils cost 1 cent each, pens cost 5 cents each and books 7 cents each. You buy 15 items totaling 75 cents. You want the number of pens to exceed the number of pencils, and you want at least one pencil. Also you want at least as many books as pens. How many pencils must you buy?

Ans: 3 pencils

7. Simplify $(A^{10} - B^{10})/(A - B)$ where $A = (1 + \sqrt{5})/2$ and $B = (1 - \sqrt{5})/2$

Ans: 55

8. Triangle **ABC** has a right angle at **B**, and contains an interior point **P** for which

PA = 10, **PB** = 6 and $\angle \text{APB} = \angle \text{BPC} = \angle \text{CPA}$. Find the length **PC**

Ans: $PC = 33$

9. Find an equation for the plane which passes thru the points **(1,6,0)**, **(2,9,1)**, **(0,-2,-2)** and **(3,7,1)**

Ans: $2x + y - 5z = 8$

10. A circle of radius r goes thru two neighboring vertices of a square and is tangent to the side of the square opposite those vertices. In terms of r the area of the square is what ?

Ans: $\frac{64}{25}r^2$

11. What is the largest integer n such that $\frac{n^3+n+164}{n^2+1}$ is an integer?

Ans: 9

12. A particular model of jet engine is used in a two engine plane and a four engine plane. The probability that one of these engines fails in flight is p . The two engine plane will crash only if both engines fail and the four engine plane will crash only if three or more engines fail. Assume engine failure is the only cause of plane crashes, and that the engines operate independently. For which values of p is the two engine plane safer?

Ans: $1/3 < p < 1$

13. A student walks from home to school, which is located 5 blocks East and 7 blocks North of his house. She always walks a total of 12 blocks, stopping to meet her friend who lives 3 blocks East and 4 blocks North of her house. How many different walks from her house to school via her friend's house are there?

Ans: $\binom{7}{3} \binom{5}{2} = 35 \bullet 10 = 350$

14. If $\sin(x) = 3 \cos(x)$ then what is $\sin(x) \cos(x)$?

Ans: $3/10$