DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes long. Questions 1-4 are each worth 1 point. Questions 5-8 are each worth 2 points. Questions 9-11 are each worth 3 points. Calculators and other electronics MAY NOT be used.

1 Point Each:

1. What are all the nonnegative integers $x$ for which $(x-6)(x + 14)$ is a perfect square?
   ans: 6, 22

2. A cow on 30’ rope is hooked to a corner of a 10’ x 20’ barn. How many square feet of grazing area does she have?
   ans: $800\pi$ ft$^2$

3. Assume that all of the following are nxn invertible matrices: $A, B, C, F, X$. Solve the following equation for $X$: $A^2B^{-1}XC + F = I_n$. ($I_n$ denotes the nxn identity matrix)
   ans: $BA^{-2}(I_n - F)C^{-1}$

4. Consider the circle $x^2 + y^2 = 16$ and the ellipse $x^2/16 + y^2/9=1$. What is the area inside the circle and outside the ellipse?
   ans: $4\pi$
2 Points Each:

5. If x is a real number such that \(2x^3 + 4x^2 + 6x + 8 = 2468\), what is the value of \(x^3 + 9x^2 + 8x + 8\)?
   \[\text{ans: } 1988\]

6. What is the area of the smallest circle centered at the origin which touches both branches of the hyperbola \(xy = 2\)?
   \[\text{ans: } 4\pi\]

7. Please evaluate
   \[
   \sum_{j=2}^{5} \sum_{i=1}^{6} (ji^2 + 2)
   \]
   \[\text{ans: } 1322\]

8. In a certain school, 60% of the students have a dog at home. Suppose that 8 students are sampled. What is the probability that exactly 5 have a dog at home? (no need to simplify your resulting answer)
   \[\text{ans: } \frac{8!}{(5!3!)} \times (0.6)^5 \times (0.4)^3\]
3 Points Each:

9. Consider the infinite series \( 1 + \frac{i}{2} - \frac{1}{4} - \frac{i}{8} + \frac{1}{16} + \frac{i}{32} \ldots \) where \( i = \sqrt{-1} \) which converges to a point in the complex plane. If the distance of that point from the origin is \( \frac{a\sqrt{5}}{b} \) find \( a \) and \( b \).  
   \[ \text{ans: } a=2 \quad b = 5 \]

10. What is the value of \( k \) for which

\[
\frac{2000!}{1000!} = k(1 \times 3 \times 5 \times 7 \ldots \times 1997 \times 1999) \\
\]
   \[
\text{ans: } k = 2^{1000} \\
\]

11. How many 0’s are at the end of \( 62! \)  
   \[ \text{ans: 14} \]