1) Simplify $1 + i + i^2 + \cdots + i^{100}$
   $i = \sqrt{-1}$
   Ans: 1

2) Simplify the following matrix product.

\[
\begin{bmatrix}
\frac{\sqrt{3}}{2} & -\frac{1}{2} \\
\frac{1}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix}^{1203}
\begin{bmatrix}
3 \\
3
\end{bmatrix}
\]

Ans: (-3,3)

3) Find positive integer solutions to

\[a^3 - b^3 - c^2 = 3abc\]
\[a^2 = 2(b + c)\]

Ans: a=2, b=1, c=1

4) Solve for x.

\[(6x + 28)^{\frac{1}{3}} - (6x - 28)^{\frac{1}{3}} = 2\]

Ans: $x = +/- 6$
5) Which one cannot be folded into a cube?

![Diagram of cube nets]

Ans: B) cannot be

6) Solve the following system:

\[
\begin{align*}
x + y + z + u &= 5 \\
y + z + u + v &= 1 \\
z + u + v + x &= 2 \\
u + v + x + y &= 0 \\
v + x + y + z &= 4
\end{align*}
\]

Ans: \(x=2, y=1, z=3, u=-1, v=-2\)

7) A system of 2 linear equations and 2 unknowns have coefficients that are in arithmetic progression. What is the solution to the system?

Ans: -1 and + 2

8) John wants to buy $1 worth of stamps. He asked for 2 cent stamps, ten times as many 1 cent stamps and the rest in 5 cent stamps. How many of each did he buy?

Ans: 50 one cent stamps, 5 two cent stamps, 8 five cent stamps
9) One of the following numbers is prime. They are displayed in binary form. Which one is prime?

   a) 10001000₂
   b) 10100110₂
   c) 10000001₀₂
   d) 10111101₁₂
   e) 10011100₀₂

   Ans: d) is prime

10) Suppose 6 students are taking an exam in a row of seats, with an aisle at each end. If they finish the exam in random order, what is the probability that a student will have to pass over one or more of the other students in order to reach the aisle?

   Ans: \(1 - \frac{2}{6} \times \frac{2}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{43}{45}\)

11) Evaluate

   \[\log_{10} \cot(1^\circ) + \log_{10} \cot(2^\circ) + \log_{10} \cot(3^\circ) + \ldots + \log_{10} \cot(87^\circ) + \log_{10} \cot(88^\circ) + \log_{10} \cot(89^\circ)\]

   Ans: 0

12) What is the 2008-th digit in the decimal representation of \(\frac{1}{14}\)?

   Ans: 4

13) In a certain high school there are 605 students. There are 40 more sophomores than freshman and one half as many freshmen as juniors. The number of seniors is 30 less than 3 times the number of freshman. How many seniors are there?

   Ans: 225 seniors

14) Solve for \(x\).

   \[(\ln(3x))^4 - (\ln(3x))^3 - 6(\ln(3x))^2 = 0\]

   Ans: \(x = 1/3, e^{3/3}, e^{-2/3}\)